

The Political Economy of
Overlapping Generations

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The formation and maintenance of the institutions of money and a futures market are analyzed in an overlapping generations model with a first period. With money and a futures market the economy converges to the allocation where costly transactions are foregone and marginal products and marginal utilities equated. However, neither institution may be formed, or money may be formed without a futures market. Moreover, stochastic output technologies raise the possibility of persistent recession and depression and of valuable government insurance of the futures market.

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The Political Economy of Overlapping Generations
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This paper presents an analysis of the formation of the institutions of money and of a futures market in an overlapping-generations model. The crucial assumption in this analysis is that there is a beginning of time, so that one cannot brush aside the problem of the development of institutions by assuming that it occurred in the infinite past. This paper does violate what are typically taken to be the legitimate bounds of economic analysis in discussing essentially political institutions. However, it seems to the author that in studying fiat money, which in practice, and possibly inherently, is a nonmarket phenomenon, one has little choice but to do so. "It is because it always was" is just not a satisfactory answer.

There are two reasons for the introduction of the nonmarket institution of a futures market in addition to the institution of money. In the first place, the two institutions bear an obvious relation to each other as the model shows. Secondly, this paper was in part motivated by the desire to produce a model in which a "banking sector" is a nonmarket phenomenon, the collapse of which could have severe and enduring effects. This is one explanation for the persistence of recession and depressions.^{1/} In the model both money and a futures market serve to allow costless exchanges substitute for costly exchanges between individuals. However, the reasons that the two institutions are non-market phenomena are opposite. The problem with initiating the money market is that it is valuable to do so, so how do you allocate the seigniorage? The problem with initiating the futures market is that it is costly to do so, so how do you impose the cost? This difference is important. If the economy "collapses" in the sense of generating low output, the institution of money can immediately recover, while the futures market cannot.

^{1/} See Bryant [1].

The analysis is performed on a particular, simple example of an economy. This approach is taken to make the analysis tractable, and because the example proves rich enough to illustrate the interesting issues.

The paper is organized as follows. First the basic structure of the model is provided. Then the polar case where the costly transactions are prohibitively costly is discussed. We turn next to the general case of costly transactions. In this section we first consider only the institution of money before adding the possibility of a futures market. Next we turn to a brief discussion of the possible collapse of the futures market, and the insurance of futures contracts. Lastly we turn to the issue of the enforcement of contracts, the institution of which is implicit in the preceding discussion.

I. The Model

The model is a version of Samuelson's [6] pure consumption loans model. Time is divided into discrete periods. N two-period lived individuals are born per period. Each individual is endowed with one unit of labor in his first period of life. There are two technologies for producing the single transferable but nonstorable consumption good. These technologies exist at physically separate sites. At site one an input of n units of labor this period produces n^β units of output per unit of input this period, $-1 < \beta < 0$. Therefore, total output is increasing, concave, and has infinite slope at zero. At site two an input of $N-n$ units of labor this period produces $(N-n)^\beta$ units of output next period per unit of input. Trade between the two sites is possible at goods cost c per unit of goods traded. $0 \leq c \leq 1$.

All individuals have the same preference ordering over consumption bundles. Let $C_1, C_2 \subset \mathbb{R}^+ \times \mathbb{R}^+$ be the individual's consumption of the consumption good in his two periods of life. The individual is indifferent between consumption bundles that differ only in regard to the labor input of the individual.

For $C_1, C_2 > 0$ his preferences are ordered by the consumption function $U(C_1, C_2) = \log C_1 + \log C_2$. Any bundle with $C_1, C_2 > 0$ is preferred to any bundle with $C_1, C_2 \in 0 \times \mathbb{R}^+ \cup \mathbb{R}^+ \times 0$. For $C_1, C_2 \in 0 \times \mathbb{R}^{++} \cup \mathbb{R}^{++} \times 0$ the bundles are ordered by

$$= \begin{cases} \log C_1, & \text{if it exists} \\ \log C_2, & \text{if it exists} \end{cases}$$

Lastly, $C_1 = C_2 = 0$ is the least preferred bundle in $\mathbb{R}^+ \times \mathbb{R}^+$. The log utility function is chosen because it is simple, income and substitution effects are exactly offsetting, and it satisfies the usual properties for guaranteeing an internal solution.

At birth individuals choose a technology. They are ordered and jump sequentially to a technology site at which they are stuck for life. Individuals who jump to site i will be referred to as type i , $i = 1, 2$. In general we will ignore the integer problem, and assume that the individuals can always equalize the return at the two sites. This can be made rigorous by having a continuum of individuals. Before they are ordered and jump the individuals have the option of holding a mass meeting. This mass meeting takes up a proportion α of their labor input $0 \leq \alpha < 1$. We assume that they all meet, or do not meet, but this seems an innocuous assumption as all individuals are identical. No collusion can occur after the individuals are ordered and jump.

II. Prohibitively Costly Trade

First we consider the case for $c = 1$, so that trade between the sites is impossible. In all that follows we consider perfect foresight equilibria.

There is a nonmonetary equilibrium. In that equilibrium individuals at site one consume only in the first period, and individuals at site two consume only in the second period. Since a core solution requires that identical individuals consume identically we conclude that at site one each individual

consumes n^β and at site two each individual consumes $(N-n)^\beta$. Moreover, the individuals will jump so as to equalize utility at each site, so $\log n^\beta = \log(N-n)^\beta$ or $n = N/2$.

We have assumed that individuals are allocated to sites by sequentially "jumping" to them by individual choice. There is a question whether this allocation device would be maintained. A coalition might be able to set up a device that would allocate individuals according to the outcome of a lottery. In such a lottery system, prelottery all individuals would be treated equally, and the coalition would impose its unequal post lottery distribution. Such a system would, at best, maximize expected utility. For the moment let us assume the more general utility function $U(C_1, C_2) = \log C_1 + \gamma \log C_2$, $0 < \gamma < \infty$. Then our original allocation device produces $\log n^\beta = \log((N-n)^\beta)^\gamma$ or $n = (N-n)^\gamma$. The best lottery allocation will solve:

$$\max_{0 \leq n \leq N} \frac{n}{N} \log n^\beta + \frac{N-n}{N} \log(N-n)^{\beta\gamma}.$$

This is solved by $n = (N-n)^\gamma e^{\gamma-1}$. This equals our above solution only at $\gamma = 1$. For $\gamma \neq 1$ our individuals prefer a risky allocation device to a riskless one. However, as we assumed $\gamma = 1$, we need not worry that individuals have a motive to meet just to change the allocation device.

Let us now consider a monetary equilibrium. As money can only be used to "transform" goods today into goods tomorrow, it will only be held at site one. Moreover, any monetary equilibrium will allow the individual to consume in both periods of life in site one, so only site one is used, $n = N$. In what follows we will use time subscripts only when necessary. From generation two on the individuals' problem is:

$$\max_z \log[N^\beta - z] + \log[\pi z]$$

where z is the goods value of the individual holding of money, and π is the ratio of the goods value of a unit of money tomorrow to the goods value of a unit of money today. This is maximized at $z = N^\beta/2$. Since individual money holdings are the same in each period we conclude that $\pi = 1$ is the only monetary solution. It should be noted that this result that the only possible monetary equilibrium is the noninflationary equilibrium is not a general result, but rather depends upon our choice of utility function.^{2/}

For the "monetary equilibrium" to indeed be an equilibrium, we must guarantee that a future generation will not find it advantageous to collude to reject generation one's money and set up its own monetary system. This is the "seigniorage problem." That is, we must have that the utility of generation one ($U(\text{generation one})$) is less than or equal to the utility of generation two, or

$$\log[(1-\alpha)^{\beta+1}N^\beta] + \log(N^\beta/2) \leq \log(N^\beta/2) + \log(N^\beta/2)$$

or

$$\alpha \geq 1 - \left(\frac{1}{2}\right)^{\beta+1} > \frac{1}{2}.$$

Therefore, a necessary condition for the monetary equilibrium to be Nash is that the collusion meeting use up more than half the individual's labor endowment. We do not have to worry about it being worth generation one's while to meet, since this is the only way they can get consumption in both periods. We have, then, the rather paradoxical result that the world with high meeting cost is Pareto superior to the world with low meeting cost if the monetary equilibrium obtains in the former.

In this world a futures market will not be set up. The first inhabitants of site two (if there are any) can get no consumption in their first period of life, and cannot be compensated for "seeding" a futures market by

^{2/}See, for example, Bryant [1].

giving up goods in their second period of life for promises of goods in the following period.

There is an alternative institution to money which must be considered. Suppose a system of costless lump sum taxes can be set up by generation one. The imposed tax sequence will solve

$$\begin{aligned} & \max t_1 \\ & t_1, t_2, \dots \\ & \text{s.t. } \log[N^\beta - t_{j-1}] + \log t_j \geq \log[(1-\alpha)^{\beta+1} N^\beta] + \log t_1, \\ & \quad j = 2, 3, \dots \end{aligned}$$

The solution is $\underline{t} = (t, t, t, \dots)$ where $\log[N^\beta - t] + \log t = \log (1-\alpha)^{\beta+1} N^\beta + \log t$, or $t = [1 - (1-\alpha)^{\beta+1}] N^\beta$, which is feasible for any α . If a monetary equilibrium is feasible then $\alpha \geq 1 - (\frac{1}{2})^{\frac{1}{\beta+1}}$ or $(1-\alpha) \leq (\frac{1}{2})^{\frac{1}{\beta+1}}$. In this case $t = [1 - (1-\alpha)^{\beta+1}] N^\beta \geq [1 - ((\frac{1}{2})^{\frac{1}{\beta+1}})^{\beta+1}] N^\beta = \frac{1}{2} N^\beta = z$. In other words, such a tax strategy is at least as beneficial to generation one as the money strategy. That a costless system of lump sum taxes can substitute for the monetary solution is, of course, well known.

Why, then, would the monetary equilibrium ever appear? One answer is that a costless system of lump sum taxes is not feasible. Moreover, the repeated cost of lump sum taxing exceeds the cost of preventing counterfeiting (which we have implicitly assumed to be zero). This argument gains force when one considers that the pure consumption-loans model is capturing the idea that not all contracts are feasible and costless, and money can help to "bridge" some of the gaps. In reality, using tax to produce optimal allocations is very complicated, and therefore likely to be a very costly procedure.

III. The General Case.

We now turn our considerations to the case where $0 < c < 1$. While in the following discussion we will assume for expositional clarity that $c \neq 0$, the

reader can go through the same arguments with $c = 0$. He will conclude that there is no monetary or futures equilibrium, that $n = N/2$, and $C_1 = C_2 = (N/2)^\beta/2$ at both sites.

First we examine the case where there is no futures market, and consider the nonmonetary equilibrium. Let d^1 be the lendings of type one individuals, d^2 be the borrowings of type two individuals and Ω the rate of return on loans. Then our problem is:

$$\text{(type one individuals) } \max_{d^1} \log[n^\beta - d^1] + \log[\Omega d^1] = I$$

$$\text{(type two individuals) } \max_{d^2} \log[(1-c)d^2] + \log[(N-n)^\beta - \frac{\Omega}{1-c}d^2] = II$$

$$\text{with } nd^1 = (N-n)d^2 \equiv D$$

$$\text{and } I = II.$$

The maximization problems are solved at

$$d^1 = n^\beta/2 \text{ and } \frac{\Omega}{1-c} d^2 = (N-n)^\beta/2.$$

By the symmetry of the problem we can conclude that $n = N/2$ and $\Omega = 1 - c$ (this can be demonstrated by a little algebra as well).

Now we turn to the monetary equilibrium without the futures market, which is a substantially more difficult problem. From generation two on our problem now can be written as:

$$\max_{z, d^1} \log[n^\beta - z - d^1] + \log[\pi(z + d^1)] = I$$

$$\max_{d^2} \log[(1-c)d^2] + \log[(N-n)^\beta - \frac{\pi}{1-c}d^2] = II$$

$$nd^1 = (N-n)d^2 = D$$

$$I = II.$$

Note that we have already imposed the result that the rates of return on money and lending must be identical for positive lending. The maximization problems are solved at $z + d^1 = n^\beta/2$ and $\frac{\pi}{1-c} d^2 = \frac{(N-n)^\beta}{2}$. Plugging these into I and II, the equality of I and II then implies that $\frac{n^\beta}{(N-n)^\beta} = \frac{1-c}{\pi}$. Defining $Z = nz$, these three equations can be rewritten as:

$$(1) \quad Z + D = n^{\beta+1}/2$$

$$(2) \quad \frac{\pi}{1-c} D = (N-n)^{\beta+1}/2$$

$$(3) \quad n^\beta/(N-n)^\beta = (1-c)/\pi.$$

Given π , these three equations uniquely determine Z , D , and n . It immediately follows that $\pi = 1$ is the only possible constant rate of inflation as any other rate also determines a unique z . Letting prime refer to the next period value, if $\pi = \pi'$ then $Z = Z'$ and $\pi = Z'/Z = 1$.

We know that the only constant inflation monetary equilibrium is the noninflationary monetary equilibrium. Is this the only possible monetary equilibrium? We will demonstrate below that this is not the case. The conclusion is that a unique and constant price monetary equilibrium is not a robust result. Our demonstration proceeds by attempting a conventional proof that the constant price equilibrium is the only monetary equilibrium.

First we show that $dZ/d\pi > 0$. Plugging (3) into (2) we get $D(N-n)^\beta/n^\beta = (N-n)^{\beta+1}/2$ or $D = n^\beta(N-n)/2$. From (1) we have then that $(n^{\beta+1}/2 - Z) = n^\beta(N-n)/2$ or

$$(4) \quad Z = (n - N/2)n^\beta.$$

totalling differentiating (4) we have $dZ/dn = (1+\beta)n^\beta - \beta n^{\beta-1}N/2 > 0$. However, totally differentiating (3) yields $dn/d\pi = \frac{-(N-n)^2}{\beta N} \left(\frac{n}{N-n}\right) > 0$. Then $dZ/d\pi = (dZ/dn)(dn/d\pi) > 0$.

Now we attempt to show that $dZ/d\pi > 0$ and $\pi \neq 1$ yields a contradiction. Let $\pi = 1$, $Z = \bar{Z}$ solve (3) and (4). Suppose in a monetary equilibrium at some period t , $\pi_t > 1$. Then $Z_t > \bar{Z}$. Then $\pi_{t+1} > \pi_t$, . . ., or Z is increasing thereafter at an increasing rate, which is impossible as $Z < 2(N/2)^\beta$. So far so good. But now suppose at some period t , $\pi_t < 1$. Then π_t is monotonically decreasing thereafter. Therefore Z approaches zero. By (4) either n approaches zero or $N/2$. Since at $Z = n = 0$, (1) contradicts (2) it must be that n approaches $N/2$. Therefore, from (3) π is decreasing monotonically to $(1-c)$. We have the possibility of inflation rising monotonically to $\frac{c}{1-c}$. Where the proof falls down is that if Z is the only form of saving, one can bound Z below, but here we can only bound $Z + D$ below as in (1). The introduction of the possibility of costly trade also introduces the possibility of inflation, but at a rate which is bounded above, with this bound depending on the cost of trade.

Next we check to see whether the noninflationary monetary equilibrium is, indeed, a Nash equilibrium. First we must show that following generations will not have motive to meet, reject generation one's money, and set up their own money. That is to say, we must as in the $c = 1$ case insure that the seigniorage does not exceed the cost of meeting, $U(\text{generation one: monetary}) \leq U(\text{generation two: monetary})$. With some algebra it can be shown that the second period consumption of a generation one individual of type one is $(1-c)(1-\alpha)^{\beta+1}(N/2)^\beta$. As the return from money "production" is a lump sum, in the first generation $n = N/2$, and the rate of return on contracts serves to share \bar{Z} with type two individuals, $\Omega = (1-c) - \bar{Z}/[(1-\alpha)^{\beta+1}(N/2)^{\beta+1}]$. Now the consumption after generation one is independent of α , so for α close enough to one $U(\text{generation one: monetary}) < U(\text{generation two: monetary})$, since $\lim_{\alpha \rightarrow 1} U(\text{generation one: monetary}) \rightarrow -\infty$.

This is not, however, sufficient for the monetary equilibrium to obtain. We must also show that the first generation is better off meeting and instituting money than not meeting and having the nonmonetary equilibrium

obtain. We need $U(\text{nonmonetary}) < U(\text{generation one: monetary}) \leq U(\text{generation two: monetary})$. We know that $U(\text{nonmonetary}) < U(\text{generation two: monetary})$. We also know that at $\alpha = 0$ $U(\text{generation one: monetary}) > U(\text{nonmonetary})$. Moreover, as $U(\text{generation one: monetary})$ is monotonically decreasing without bound in α , we know that there is a feasible range of α . So the noninflationary monetary equilibrium can obtain for a range of α not so low as to make the seigniorage attractive to future generations, nor so high as to make seigniorage unattractive to the first generation.

So far we have ignored the possibility of type two individuals of generation one "seeding" a futures market. Type two individuals of generation one could give up goods to type two individuals of generation two in return for promises to goods the next period. Now generation one individuals have no use for such promises. However, when individuals of generation two type two deliver on their promises, individuals of generation three type two can buy the goods with promises of goods tomorrow, and so on. As with money, we assume that maintaining this futures market institution is costless. With this institution type two individuals can be maintained without costly trade with type one individuals. But why would generation one set up such an institution? They would do so only if it would increase generation two's demand for money enough so that generation one type one individuals would subsidize generation one type two individuals for "seeding" the market. Our task is to show if and when such an institution would be set up, and what its consequences would be.

First we examine the consequences of such a futures institution by examining this problem of generation two and following generations. Let q be the number of goods that a type two individual buys in the futures market with promises of Pq units of goods next period. Then the problem from generation two on can be written:

$$\max_{z, d^1} \log[n^{\beta-z-d^1}] + \log[\pi z + \Omega d^1] = I$$

$$\max_{d^2, \ell} \log[(1-c)d^2 + \ell] + \log[(N-n)^{\beta-P\ell - \frac{\Omega}{1-c} d^2}] = II$$

$$\text{with } nd^1 = (N-n)d^2$$

$$(N-n_t)\ell_t = (N-n_{t-1})P_{t-1}\ell_{t-1} \quad t = 2, \dots$$

and I = II.

The maximization problems are solved at $z + d^1 = n^{\beta}/2$ and $P\ell + \frac{\Omega}{1-c} d^2 = \frac{(N-n)^{\beta}}{2}$. The maximization problems also imply $\pi \geq \Omega$, = if $d^1 > 0$; $P \leq \frac{\Omega}{(1-c)^2}$, = if $d^2 > 0$. Plugging into I and II, the equality of I and II implies $\frac{n^{\beta}}{(N-n)^{\beta}} = \frac{1}{\sqrt{\pi P}} \geq \frac{1-c}{\pi}$, = if $d^1, d^2 > 0$.

Defining $L = (N-n)\ell$ and remembering that $Z = nz$, $D = nd^1 = (N-n)d^2$ we have

$$(5) \quad Z + D = n^{\beta+1}/2$$

$$(6) \quad PL + \frac{\Omega}{1-c} D = (N-n)^{\beta+1}/2$$

$$(7) \quad \frac{n^{\beta}}{(N-n)^{\beta}} = \frac{1}{\sqrt{\pi P}} \geq \frac{1-c}{\pi} = \text{if } D > 0.$$

Let us examine our equations. First suppose that the solution has $D > 0$ for all t . Further suppose $L_2 > 0$. Then

$$L_t = P_{t-1}L_{t-1} = \frac{\pi_{t-1}}{(1-c)^2} L_{t-1} = \frac{Z_t}{Z_2} \frac{1}{(1-c)^{2(t-2)}} L_2.$$

Therefore, Z_t approaches zero at least at the rate $(1-c)^2$ as L is bounded above. Conversely if Z_t is bounded below then D must be zero after a finite number of periods.

Now we consider such a monetary equilibrium where Z is bounded below.

To do this we examine equations (5)-(7) with $D = 0$. (5) implies that $\pi_t = (\frac{n_{t+1}}{n_t})^{\beta+1}$. $L_t = P_{t-1}L_{t-1}$ and (6) imply $P_t = (\frac{N-n_t}{N-n_{t-1}})^{\beta+1}$. Plugging this into (7) and rearranging yields

$$n_t = \frac{1}{\frac{N-n_{t-1}}{n_{t+1}} \frac{1+\beta}{1-\beta} + 1} N.$$

Defining δ_t by $n_t = (1+\delta_t)N/2$ we have

$$(1+\delta_t)N/2 = \frac{1}{\frac{1-\delta_{t-1}}{2} \frac{1+\beta}{1-\beta} + 1} N$$

or

$$(8) \quad \delta_t = \frac{2}{\frac{1-\delta_{t-1}}{1+\delta_{t+1}} \frac{1+\beta}{1-\beta} + 1} - 1.$$

Since $0 < \frac{1+\beta}{1-\beta} < 1$, it follows that $|\delta_t| < \max\{|\delta_{t-1}|, |\delta_{t+1}|\}$. But this implies that $|N/2 - n_t|$ must be either nondecreasing or nonincreasing for all t . Suppose it is nondecreasing. Then $|\delta_t|$ is a nondecreasing sequence bounded above (by $N/2$). Therefore, $|\delta_t|$ has a limit. But it follows from (8) (plugging in $\delta_t = \delta_{t-1} = \delta_{t+1}$) that $|\delta| = 0$ is the only possible limit point. Similarly if $|N/2 - n_t|$ is nonincreasing. We conclude that n_t approaches $N/2$. Therefore, a monetary equilibrium with Z strictly bounded away from zero approaches the solution $n = N/2$, $z = (N/2)^{\beta/2}$, $\lambda = (N/2)^{\beta/2}$, $D = 0$, and $C_1 = C_2 = (N/2)^{\beta/2}$ for both types of individuals.

For any $L_2 > 0$ the economy can achieve in the limit the "optimal" allocation of individuals equally divided between the two technologies, consuming equally in both periods and engaging in no costly trade. But will generation one choose $L_2 > 0$?

In the first place, type one individuals of generation one can be given less in payment on their loans to type two individuals to offset any gain in the value of money. We conclude that for $dZ_2/dL_2 > 1-c$ increasing L_2 is worthwhile for generation one (this argument can easily be made rigorous). Moreover, for $Z, D > 0$ we have from (5)-(7) $Z + D = n^{\beta+1}/2$, $\frac{\pi}{(1-c)^2} L + \frac{\pi}{1-c} D = (N-n)^{\beta+1}/2$, and $n^\beta/(N-n)^\beta = (1-c)/\pi$. If we take $\pi = 1$, then $dZ/dL|_{\pi=1} = -dD/dL|_{\pi=1} = -(-\frac{1}{1-c}) = \frac{1}{1-c} > 1-c$. Consider the noninflationary monetary solution with $L = 0, Z = \bar{Z}$. Now if $Z_2 = \bar{Z} + \frac{L}{1-c}$ holding π fixed at one implies $Z_t = Z_2 = \bar{Z} + \frac{L}{1-c}, t \geq 2$. But we know that Z_t approaches $(N/2)^{\beta+1}/2$ for $L_2 > 0$. Therefore, if $\bar{Z} < (N/2)^{\beta+1}/2$ for L_2 small we have not overstated dZ/dL by failing to consider falling π . So if $\bar{Z} < (N/2)^{\beta+1}/2$ generation one will set $L_2 > 0$.

What can be said about $\bar{Z}/(N/2)^{\beta+1}/2$? \bar{Z} satisfies $n^\beta/(N-n)^\beta = 1-c$ and $\bar{Z} = (n-N/2)n^\beta$, (3) and (4). Let $Z^* = (N/2)^{\beta+1}/2$. From (4) we conclude that $\bar{Z}/Z^* = \frac{1}{(N/2)^{\beta+1}} [2n^{\beta+1} - n^\beta N]$. Define λ by $n = \lambda N/2$. Plugging this into the previous expression and rearranging yields $\bar{Z}/Z^* = 2\lambda^\beta [\lambda - 1]$. Substituting λ into (3) yields $\lambda = \frac{2(1-c)^{1/\beta}}{1+(1-c)^{1/\beta}}$. These two equations yield

$$(9) \quad \bar{Z}/Z^* = \frac{[(1-c)^{1/\beta}]^{\beta+1} - (1-c)^{1/\beta} 2^{\beta+1}}{[1+(1-c)^{1/\beta}]^{\beta+1}}$$

This implies that for given c , \bar{Z}/Z^* can be anywhere in the interval $(c, 2)$ depending upon β and for given β can be anywhere in the interval $(0, 2^{\beta+1})$ depending upon c . Setting $L_2 > 0$ could actually cut the limiting value of money almost in half. On the other hand, for some choices of c and β , \bar{Z}/Z^* can be very small, and generation one does set $L_2 > 0$. Will generation one ever set $L_2 = (N/2)^{\beta+1}/2$ so that the economy moves at once to the "optimal" allocation? No, for at $L_2 = Z^*$ the return to generation one on L_2 is

$$\frac{Z^* - \bar{Z}}{L_2} = \frac{Z^* - \bar{Z}}{Z^*} = 1 - \frac{\bar{Z}}{Z^*} < 1-c.$$

This result is not surprising. At $D = 0$ (5)-(7) become $Z = n^{\beta+1}/2$, $PL = (N-n)^{\beta+1}/2$, $n^{\beta}/(N-n)^{\beta} = 1/\sqrt{P\pi}$. Differentiating this system yields $dZ/dL|_{\pi=1} < 0$.

The economy will not move to the "optimal" allocation immediately. For some parameters values it can converge to the "optimal" allocation. However, for some values of c and β generation one will not set $L_2 > 0$, although any amount of L_2 allows the economy to converge to the "optimal" allocation. We have a solution that any social welfare function with interpersonal comparisons of utility and positive weights on all generations would find suboptimal. Note that the possibility that $L > 0$ does not change our argument that the monetary equilibrium is indeed Nash, although the set of α for which this is so may change, as setting up both money and futures markets may be more attractive than setting up the money market alone.

So far we have not considered the possibility of generation one taxing to recover the costs of "seeding" the futures market. This is because we have already assumed that taxing to generate an optimal allocation of goods is costly. However, one could "finance" the setting up of a futures market by a one-time tax on type one individuals of generation two. Suppose such a one-time tax has zero cost given that generation one has met. Then generation one will set $L_2 > 0$ and finance it with such a tax if necessary, as this is a way to substitute costless for costly transfers.

IV. Collapse of the Futures Market

We now wish to briefly consider what happens if the production technologies are very occasionally and unpredictably subject to independently distributed downward shocks. If it is the type one technology that is hit by a shock in period t , then type one generation t individuals will pay less for money, but will sell it to the $t+1^{\text{st}}$ generation at an unchanged price. The

economy simply continues where it left off. Even if the t^{th} generation type one's get zero output, they will desire the money, and the $t-1^{\text{st}}$ generation will be willing to part with it at zero price. In contrast, if the type two technology is "hit," the effects are of longer duration. Fewer $t+1^{\text{st}}$ generation individuals will be of type 2, and the economy will then converge back to the "optimal allocation." If the type two's get zero output the futures' market collapses, and will not regenerate itself. Indeed, it may not be worth it for future generations to bear the cost of meeting just to set up the futures market, and the economy is permanently reduced to the $Z = \bar{Z}$, $L = 0$ solution.

However, the question cannot just be left here. If it is not too costly to maintain continuously and to finance when needed, generation one may set up a system of insuring individuals against a bad return to a technology. There is good reason why optimal insurance may not be provided privately. Insurance contracts can be written only by individuals who are simultaneously alive before the bad outcome is realized. For example, if the bad outcome is known as soon as individuals get to the technology and before they can communicate, they are just out of luck without a "government" insurance system. Even if they can sign contracts before the outcome is known, future generations cannot be signed into a "string" of subsidies following a bad outcome. However, since "government" insurance would be financed by costly taxes, such multigeneration schemes of government insurance also would be of limited size.

One simple insurance device is for the "government" to guarantee type two individuals' promises to deliver. For some specification of the tax financing of the insurer's payout the economy will simply stay at the "optimal" allocation following a shock, although there is no reason that this should in general be the optimal insurance scheme. A bankruptcy law is another simple means of insuring the type two individuals against the bad outcome. However, it may be unnecessary as the promises to pay traded in the futures market may be

made contingent upon the outcome of the technology. In addition, there needs to be a guarantee that the futures market will continue to exist in order to protect future generations interests. A simple bankruptcy law does not do this, and type two individuals have no motive to issue the appropriate contingent futures contracts once they are at site two.

Naturally, insurance schemes private or "government" may be limited by moral hazard problems. Moreover, the "government" insurance schemes are limited by generation one's knowledge of the stochastic structure of the technologies. If generation one is not omniscient, the insurance schemes may never be put into place. Or they may be put into place only after a particularly bad outcome makes it worthwhile for a future generation to meet and do so. Or they may be put into place only as a by-product of a future generation meeting for some other purpose.

V. Plundering and Incarceration

So, far we have assumed that contracts are costlessly enforced. This seems an unreasonable assumption on the surface. Clearly type one individuals have motive to counterfeit money, and type two individuals have motive not to meet their contracts. More generally, individuals have motive to take each other's output. We now introduce technology which does result in costless enforcement of contracts.

First we introduce the technology of plundering. An individual can take goods from another against his will, but the stealer receives less goods than he takes from the individual. Plundering is a costly activity. Indeed, given the possibility of plundering, the only Nash equilibrium is for everyone to consume nothing.

Secondly, we assume an incarceration technology. There exists a technology for an individual taking another individual's goods and also keeping him from stealing or consuming in that and any future period. However, the cost

of incarceration substantially exceeds the value of any goods seized. The only Nash equilibrium is still zero consumption.

Thirdly, let us suppose that setting up and maintaining a system of rules enforced by the threat of incarceration is costless. The only costs are the normal incarceration cost when a violation occurs, and the cost of meeting to set up the rules. Under these circumstances generation one will meet to set up the rules against plundering and other contract violations enforced by incarceration. Otherwise zero consumption results. Moreover, once it has set up the system, no one will violate the rules, so the system is costless.

With the current set up of our model, as generation one definitely meets it also initiates any of the other institutions we have discussed which are feasible. We need not worry, for example, that setting up the institution of money is not worth the meeting cost for generation one, although we still do need to worry that it is worthwhile for future generations to do so.

This assumption on plundering and incarceration is not entirely innocuous, however. The first generation could decide to tax (if its cheaper than plundering) part of the next generation's output. The second generation would either have to put up with this, or be reduced to nothing in their first period, as incarcerating the entire first generation is impossible. There is a third possibility. The first generation may not have the option of playing Stackleberg. The second generation could "call their bluff" by instituting an anti-tax incarceration or plundering rule. A member of generation one trying to collect the tax would then back down. Realizing this, generation one only imposes a tax small enough that generation two is indifferent to paying it or meeting to reject generation one's institutions, and similarly for all future generations. Everyone is taxed to the point of revolt as discussed in Section II. A fourth possibility is the one we have assumed, that taxing, or plundering,

is so costly that each generation is better off with the institutions of money and futures, and without taxing or plundering.

VI. Concluding Comments

The main conclusion to be drawn from the preceding exercise is that one can model a world where the nonmarket institutions of money and a futures market will be initiated and maintained. They need not be the unexplained product of the mythical infinite past maintained by an unexplained adherence to convention. Moreover, in such a model the economy can converge to the "optimal allocation" where costly transactions are foregone, marginal products are equated between technologies, and marginal utilities equated. However, we also have observed that this happy circumstance may not occur. Neither institution may be instituted, or money may be instituted without the futures market.

There have been important by-products of this investigation. We have found that risky technology raises the possibilities of enduring real effects from shocks to the futures market, and valuable "government" insurance. This has obvious application in the study of recession, depression, and the banking system. We have also found that an inflationary monetary equilibrium is possible in an otherwise stationary world even with carefully chosen utility functions. That the only monetary equilibrium is the noninflationary one is not a robust result.

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