Federal Reserve Bank of Minneapolis
Research Department

A Fortran Subroutine for Efficiently Computing HP-Filtered Time Series

Finn E. Kydland and Edward C. Prescott*

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*Both, Federal Reserve Bank of Minneapolis. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

In previous research, we and other have represented time series as the sum of a smooth series and another series. The smooth series $\{t_i\}_{i=1}^n$ associated with a given time series $\{y_i\}_{i=1}^n$ is the solution to the problem:

$$\min_{\{t_i\}_{i=1}^n} \sum_{i=1}^{n}(y_i - t_i)^2 + s\sum_{i=1}^{n-1}[(t_i - t_{i-1}) - (t_{i+1} - t_i)]^2.$$

One method of solution is to solve the system of n linear equations which constitutes the first-order conditions for this convex programming problem. With this method, computing time and storage requirement are large if n is large.

This subroutine efficiently solves the program. Storage costs and computational time are proportioned to the number of observations n.

In the limit, the t's are the solution of the fourth-order difference equation:

$$t_{n+2} - 4t_{n+1} + (6+s)t_n - 4t_{n-1} - t_{n-2} = y_n/s.$$

This difference equation can be solved to yield doubly-infinite sequences

$$t_n = \sum_{j=-\infty}^{\infty} a_{|j|}y_{n+j}.$$

where the a's depend on the value of the smoothing factor s. The deviation series $d_i \equiv y_i - t_i$ is a time-invariant symmetric linear filter. King and Rebelo (1989) have shown that any stochastic process which is stationary under fourth differencing is stationary under this filter.

The deviations do not change if a linear trend is added to y_i. This is true for all sample sizes.

For quarterly aggregate data setting $s = 1600$ mimics well the trend curve which we, and we think most others, would draw through series. For annual aggregate data a smaller value of s is appropriate. A reasonable value is $s = 100$.

The parameter s is a smoother parameter. As s approaches infinity, the smooth component converges to the least squares linear trend.

References

Hodrick, Robert J., and Edward C. Prescott (1980),"Post-War U.S. Business Cycles: A Descriptive Empirical Investigation," forthcoming *Journal of Money, Credit, and Banking.* Discussion Paper 451, Northwestern University.

King, Robert, and Sergio Rebelo (1989), "Law Frequency Filtering and Real Business Cycles," manuscript, University of Rochester.