Discussion by José Alexandre Scheinkman*

1. Introduction

My purpose here is not to provide a general discussion of Wallace's paper in this conference but to concentrate on two points raised by him to which I think I have something to contribute.

The first of these points concerns what Wallace calls the "tenuousness of equilibria in which fiat money has value." In an overlapping generations model of fiat money, the equilibria in which fiat money has value (also called *monetary* equilibria) coexist with equilibria in which fiat money has no value (*nonmonetary* equilibria), and there are usually many monetary equilibria in which the economy behaves asymptotically, as in the nonmonetary equilibria. Furthermore, there seems to be no mechanism to insure that even small perturbations would not lead the economy into one of the class of monetary equilibria that converge toward a nonmonetary equilibrium. Wallace claims that "tenuousness is an implication of the two defining properties of fiat money, inconvertibility and intrinsic uselessness, and not of the overlapping generations friction."

In particular, he dismisses the criticism that such behavior is a consequence of the lack of a medium-of-exchange role for money in such models. Part of this note concentrates on examining this claim. In section 2 I develop a model in which money has a role as a medium of exchange. By comparing the overlapping generations model with it and with results known in the case in which real balances are assumed to enter directly in the utility function of agents, I show that tenuousness of equilibria in which fiat money has positive value seems to be related to the possibility of the economy operating in the absence of fiat money. The mathematical conditions that insure the absence of tenuousness are very similar in all three classes of models. While section 2 develops a model of money as a medium of exchange, section 3 studies more closely the issue of tenuousness in the overlapping generations model and in models where real balances enter the utility function.

The second point I discuss is the issue of government bonds. In a two-period model, as Wallace correctly points out, if money has a positive price there cannot exist any asset with a distribution of rates of return that domi-

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nates money. This observation is then used to derive implications for the role of government bonds in macro models. I point out in section 4 how these results are no longer valid in models in which one treats individuals that live for more than two periods.

As mentioned above, this comment is restricted to just a few of the points raised by Wallace. In particular, nothing is said here about the interesting implications of the model, developed in sections 1.4 and 3 of the paper, concerning what certain statistical relationships really mean (not much!). In general, I find this a provocative and interesting piece that should start some good discussions.

2. A Model of Money as a Medium of Exchange

2.1. *Necessary Conditions for Equilibrium*

In his paper at this conference, Lucas presents a Clower-type model that yields a unique equilibrium in which money has a positive value. This is obtained by requiring agents to sell their goods for money one period ahead in order to obtain other goods. If agents cannot survive with their own endowment, a unique equilibrium obtains and money has a positive value. Let us consider, however, a generalization of the model in which agents may barter as well as exchange goods for money and money for goods as in the Clower type of model.

Each period individual consumers receive y units of one of a list of nonstorable commodities. They want to consume a mix of goods with fixed relative prices. They can trade their endowment in two markets. In one they can sell in exchange for fiat money and buy the composite good with fiat money carried over from the previous period. In the other (the barter market) they can exchange part of their endowment for the composite good; but if they deliver $\bar{y}$ to that market, they get $g(\bar{y}) > 0$ units of the composite good, where g is a strictly concave function with $g(0) = 0$, $g'(0) = 1$.[1] Notice that we can rationalize the fixed relative prices in the same way as Lucas did in his conference paper. We could also view our model as one in which labor can be sold for fiat money or used directly in home production of goods, provided home production is less efficient.

Formally, a consumer solves

$$\text{(1)} \qquad \max \sum_{t=0}^{\infty} \delta^t u(c_t)$$

subject to

$$c_t = \frac{m_t - (m_{t+1} - h_{t+1})}{P_t} + y$$

provided $yP_t \leq m_{t+1} - h_{t+1}$ and

$$c_t = \frac{m_t}{P_t} + g\left(y - \frac{m_{t+1} - h_{t+1}}{P_t}\right)$$

[1]The condition $g'(0) = 1$ is convenient from the mathematical viewpoint since it preserves the smoothness of the trading opportunities. In general, one wants $g'(0) \leq 1$.

if $yP_t \geqslant m_{t+1} - h_{t+1}$, where m_0 is given, $c_t \geqslant 0$, $m_t \geqslant 0$, $m_t - h_t \geqslant 0$, and h_t is government transfers at time t, m_t the amount of money the consumer holds at t (post transfers), and P_t the price of the composite good in terms of money.[2] The utility function u is c^2 and concave.

By defining $f(z) = z$ for $z \leqslant 0$ and $f(z) = g(z)$ for $z \geqslant 0$, we may rewrite (1) for $(m_0,\{h_t\},\{P_t\})$ as

$$\max \sum_{t=0}^{\infty} \delta^t v(m_t,m_{t+1},t) \tag{2}$$

where

$$v(m_t,m_{t+1},t) = u\left[\frac{m_t}{P_t} + f\left(y - \frac{(m_{t+1} - h_{t+1})}{P_t}\right)\right]$$

subject to

$$m_t \geqslant 0, \frac{m_t}{P_t} + f\left(y - \frac{m_{t+1} - h_{t+1}}{P_t}\right) \geqslant 0$$

$$m_t - h_t \geqslant 0.$$

A necessary condition for optimality is given by

$$\frac{-u'(c_{t-1})}{P_{t-1}} f'\left(y - \frac{m_t - h_t}{P_{t-1}}\right) + \frac{\delta u'(c_t)}{P_t} \gtreqless 0 \tag{3}$$

with ($\leqslant$) if $c_t > 0$ and ($\geqslant$) if $m_t > 0$, $m_t - h_t > 0$, $t = 1,2,\ldots$.

Given a sequence of money supplies $\{\overline{m}_t\}_{t=0}^{\infty}$ with $\overline{m}_t > 0$, an equilibrium is a sequence $\{P_t\}_{t=0}^{\infty}$ such that $\overline{m}_t$ is a solution to (2) where $\bar{h}_t = \overline{m}_t - \overline{m}_{t-1}$.

In an equilibrium $c_t \geqslant g(y)$ and since $\overline{m}_t > 0$, $\overline{m}_{t-1} = \overline{m}_t - h_t > 0$, we must have

$$\frac{u'(c_{t-1})}{P_{t-1}} f'\left(y - \frac{\overline{m}_{t-1}}{P_{t-1}}\right) = \frac{\delta u'(c_t)}{P_t} \tag{4}$$

where $c_t = (\overline{m}_t/P_t) + f[y - (\overline{m}_t/P_t)]$.

2.2. *The Structure of the Equilibrium Set*

We will restrict ourselves to the case of a fixed money supply, which illustrates most of the properties of the model. In particular, we will show how this model shares the tenuousness property with the overlapping generations model. We start by fully characterizing the equilibria.

LEMMA. *A sequence $\{P_t\}_{t=0}^{\infty}$ is an equilibrium if and only if $\overline{m}_t \equiv \overline{m}$ solves (4) and*

[2] In order to save in notation, we have stated the consumers' problem already requiring them not to use the barter market whenever they want to take for the future at least as much as they get when they sell in the money market all of their initial endowment. The reader can see that this is in fact optimal by simply observing that $g(x) < x$.

(5) $$\lim_{t\to\infty} \delta^t \frac{u'(c_t)}{P_t} = 0.$$

Proof (Necessity). We know that (4) is a necessary condition. Now let $V_s(x)$ denote the value function of a consumer who starts at time s with an initial money endowment of x and faces prices $\{P_t\}_{t=s}^{\infty}$ and transfers $h_t \equiv 0$, $t=s$, $s+1, \ldots$.

Since $\{P_t\}_{t=0}^{\infty}$ is an equilibrium, $m_t \equiv \overline{m}$ solves the optimization problem of the consumer. Since $\overline{m} > 0$, the result of Benveniste and Scheinkman (1979)[3] implies that V_t is differentiable at $\overline{m}$ and furthermore

$$V_t'(\overline{m}) = \delta^t \frac{u'(c_t)}{P_t} f'\left(y - \frac{\overline{m}}{P_t}\right) = \delta^{t+1} \frac{u'(c_{t+1})}{P_{t+1}}.$$

Also, $V_t(\alpha\overline{m}) \geqslant \Sigma_{s=t}^{\infty} \delta^s u[g(y)]$. Thus $\lim_{t\to\infty} \inf V_t(\alpha\overline{m}) \geqslant 0$. Furthermore, $\lim_{t\to\infty} \sup V_t(\overline{m}) \leqslant 0$, since $V_t(\overline{m}) \leqslant \Sigma_{s=t}^{\infty} \delta^s u(y)$. Thus, for $0 < \alpha < 1$, the concavity of V_t implies that

$$V_t(\alpha\overline{m}) - V_t(\overline{m}) \leqslant (\alpha - 1)\overline{m}\,\delta^{t+1} \frac{u'(c_{t+1})}{P_{t+1}} < 0.$$

Thus,

$$\lim_{t\to\infty} (\alpha - 1)\overline{m}\,\delta^{t+1} \frac{u'(c_{t+1})}{P_{t+1}} = 0.$$

Proof (Sufficiency). This follows from the fact that $\overline{m}$ satisfies the Euler equation and the transversality condition (5) by a standard argument. (See, for example, Benveniste and Scheinkman 1976.)

Q.E.D.

It is now immediate that if there exists $\overline{P}$ *with* $f'(y - \overline{m}/\overline{P}) = \delta$ then such $\overline{P}$ is an equilibrium. The strict concavity of g guarantees that such $\overline{P}$, if it exists, is unique. We will assume its existence since otherwise no monetary equilibria will exist. Furthermore, there is no equilibrium in which $c_t \equiv y$. For if there were such an equilibrium, then from (4) we would have $P_t = \delta P_{t-1}$, and (5) would be violated.

In order to study other equilibria, let us define $\sigma(P) = P/u'(c)$ where $c = (\overline{m}/P) + f[y - (\overline{m}/P)]$. Thus (4) may be rewritten as

(6) $$\sigma(P_t) f'\left(y - \frac{\overline{m}}{P_{t-1}}\right) = \delta\,\sigma(P_{t-1}).$$

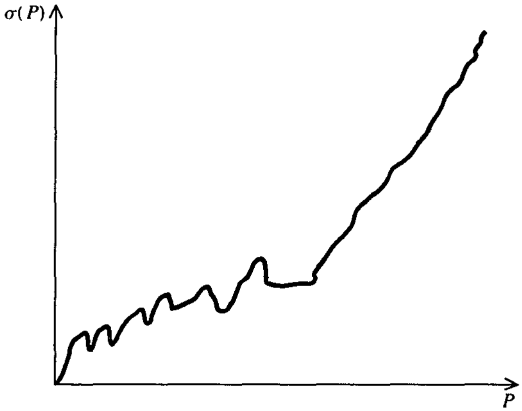
Notice that since u is C^2, σ is C^1 and

$$\sigma'(P) = \left\{u'(c) + \frac{\overline{m}u''(c)}{P}\left[1 - f'\left(y - \frac{\overline{m}}{P}\right)\right]\right\}\frac{1}{[u'(c)]^2}.$$

[3]Author names and years refer to the works listed at the end of this book.

Thus $\lim_{P\to\infty} \sigma'(P) = 1/u'[f(y)] > 0$, since $f(y) > 0$.

The figure shows the general properties of σ. Notice that $\lim_{P\to 0} \sigma(P) = 0$. Since for large P, $f'[y - (1/P)] < \delta$, it is obvious from the figure that if we choose $\bar{P}$ large enough we may obtain a solution to (6) such that $\lim_{t\to\infty} P_t = \infty$ and P_0 is any real number $\geqslant \bar{P}$. From the lemma, this is an equilibrium. Along such equilibria we have $\lim_{t\to\infty} c_t = f(y)$. That is, the economy becomes demonetized in the limit.



The Clower-type model discussed by Lucas corresponds to the case where $f(z) = 0$, $z \geqslant 0$. Even in this limit case we could still get equilibria in which the economy becomes demonetized if the consumer has utility for and can survive with the initial endowment. Thus only in the case where money is essential does this phenomenon disappear.

The model discussed in this section does have some implications which are different from those of the overlapping generations model that Wallace treats. One example consists of the optimal quantity of money which in this model is any policy that drives out barter from the system and thus would require a contraction of the money supply.[4] But in regard to the issue of tenuousness, this model does have a lot in common with the overlapping generations model, as I will show in the next section.

3. Tenuousness in the Overlapping Generations Model and in Models Where Real Balances Enter the Utility Function

Tenuousness can be avoided even in an overlapping generations model if one makes special assumptions about endowments, technology, and utility functions. In a pure trading model, a sufficient condition for this is that the endowment of old agents be zero and, if the utility function can be written as $u(c_1,c_2) = u(c_1) + \delta u(c_2)$, that

$$(7) \qquad \lim_{x\to 0} xu'(x) > 0.$$

This condition turns out also to be necessary for a large class of models

[4]This conclusion depends on $g'(0) = 1$.

(compare Brock and Scheinkman 1977). The interpretation of (7) is that traders badly need to trade consumption when young for consumption when old, and since money is the only way to do so this will avoid equilibria in which real balances converge to zero. Those are precisely the equilibria in which the economy becomes asymptotically demonetized. It is interesting to notice that the analogue of (7) is also needed to eliminate equilibria in which real balances converge to zero in models where real balances directly enter the utility function of agents (compare Brock 1978). Thus tenuousness seems to be related to how much importance fiat money has in the operation of the economy. Another way to see this point is to see what (7) implies about the inflation tax collected along the stationary equilibria as the rate of inflation goes to infinity. The first-order condition written in real balance form is

$$u'(w_y - x_\mu) = \frac{\delta}{1+\mu} u'(x_\mu) \tag{8}$$

if w_y is the endowment when young (endowment when old is zero), μ is the rate of creation of money, and x_μ is the real balances associated with the equilibrium which is stationary in real balances. Equation (8) may be rewritten as

$$\frac{u'(w_y - x_\mu)}{x_\mu u'(x_\mu)} = \frac{\delta}{x_\mu + \mu x_\mu}. \tag{9}$$

Since $\lim_{\mu\to\infty} x_\mu = 0$, (7) holds if and only if $\lim_{\mu\to\infty} \mu x_\mu > 0$, that is, if the inflation tax collected along stationary equilibria as the rate of creation of money goes to infinity is bounded away from zero. Brock (1978) shows the same point in a model with real balances in the utility function. The condition $\lim_{\mu\to\infty} \mu x_\mu > 0$ also has the interpretation that no matter how expensive it becomes to hold money people still hold a large quantity of it; that is, money is very necessary to the system. Since inconvertible fiat money seems only to appear in economic systems in which the division of labor has led to tremendous costs to pure barter, it may well be that assumptions such as (7) or (9) are not unnatural in a highly aggregated model.

4. The Role of Government Bonds

Wallace notices that in the context of his model, if fiat money has value in an equilibrium, then in that equilibrium there cannot be any asset with a rate of return distribution which dominates fiat money. This conclusion does not continue to hold if we consider a model in which agents live for more than two periods. For consider a tree that matures in two periods, and suppose there are very high costs in transacting in one-period-old trees. Then agents may hold trees in order to trade consumption when old for consumption when young but hold money in order to consume in middle age. Martins (1975) considers a model in which two-period-old bonds are issued and transaction costs on one-period-old bonds are infinite. In his model, bonds are much like another type of money and the nominal interest rate on bonds is positive and determined by the supply of bonds relative to the supply of money.

Thus Bryant and Wallace's (1979a) point that the private sector must incur transaction costs to offset the nominal interest rate on bonds is invalidated in models with more than two periods.