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A Model of TFP

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ABSTRACT

This paper proposes an aggregative model of Total Factor Productivity (TFP) in the spirit of Houthakker (1955-1956). It considers a frictional labor market where production units are subject to idiosyncratic shocks and jobs are created and destroyed as in Mortensen and Pissarides (1994). An aggregate production function is derived by aggregating across micro production units in equilibrium. The level of TFP is explicitly shown to depend on the underlying distribution of shocks as well as on all the characteristics of the labor market as summarized by the job-destruction decision. The model is also used to study the effects of labor-market policies on the level of measured TFP.

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1 Introduction

This paper focuses on the theory underlying the aggregate production function and shows how labor-market policies can affect this function in general and the level of measured Total Factor Productivity (TFP) in particular. Specifically, I construct an aggregative model of TFP in the spirit of Houthakker (1955-1956): the basic idea is to derive an aggregate production function by aggregating across active production units. In equilibrium, the levels of output, inputs and TFP as well as the shape of the aggregate relationship between them depend on individual production decisions – such as which production units remain active in the face of idiosyncratic shocks – and these decisions are in turn affected by policies. So the model can be used to study the precise interaction between all these variables explicitly.

In the model proposed here, policy affects TFP because the latter is related to the average productivity of the units which are active, and policy induces changes in the productivity composition of active units. By distorting the way in which individual production units react to the economic environment, labor-market policies can make an economy exhibit a low level of TFP. As a result, two economies may exhibit different levels of TFP even if production units in both have access to the same technology and are subject to identical shocks. In this sense the determinants of TFP levels analyzed here are different from the barriers to technology adoption of Parente and Prescott (1999, 2000).

At a theoretical level the paper also shows that, under some conditions, a standard search model of the labor market – with its underlying meeting frictions and simple fixed-proportions micro-level production technologies – can generate an aggregate production function that looks just like the one implied by the textbook neoclassical model of growth in which firms have access to a standard constant-returns Cobb-Douglas production technology. So in this sense, from the perspective of aggregate output, inputs and productivity, the neoclassical and the search paradigms can seem quite close. However, the search model implies a different mapping between the parameters of the aggregate production technology and observables, and this can have significant implications for growth accounting.

At a conceptual level, the paper is related to the vast literature that documents and tries to explain differences in TFP levels across countries. Examples include Hall and Jones (1999), Klenow and Rodríguez-Clare (1997), and Parente and Prescott (2000). This body of work has established that differences in TFP account for a large fraction of the variation in output per worker between the world's richest and poorest countries. And in terms of explanations, it shares the basic idea that the level of an economy's TFP is determined by the quality of its "institutions." Hall and Jones (1999) argue that differences in observed TFP are driven by differences in the institutions and government policies they collectively refer to as "social infrastructure." Corrupt government officials, severe impediments to trade, poor contract enforcement and government interference in production are some of their examples of bad "social infrastructures" that could lead to low levels of TFP.

Parente and Prescott (1994) propose that some countries have lower TFP than others because their process of technology adoption at the micro level is constrained by "barriers to riches." These barriers are essentially any institution or government policy that increases the cost of technology adoption. From that perspective, this paper can be thought of as adding labor market policies to the list of "institutions" that affect the level of TFP.[1] The paper is also related to the large body of work that tries to account for differences in macroeconomic performance – usually employment rates and labor productivity – with differences in labor market policies – typically unemployment benefits or employment protection. Examples include Hopenhayn and Rogerson (1993), Marimon and Zilibotti (1999), Millard and Mortensen (1997), Restuccia and Rogerson (2004) and Veracierto (2001).

[1] Unlike Hall and Jones (1999), Parente and Prescott (2000), or Acemoglu and Zilibotti (2001), this paper is not about development economics, in the sense that it does not attempt to explain why some countries are 30 times richer than others in per-capita terms. The emphasis on labor market institutions makes the analysis more relevant to study productivity differences among a relatively homogeneous set of countries (or sectors). For example, Hall and Jones (1999) and Klenow and Rodríguez-Clare (1997) report productivity differences among OECD countries, with France and Italy having a higher and Germany a much lower level of TFP relative to the United States. These obserations may seem striking, especially given the conventional wisdom that France and Italy have distorted labor markets vis-á-vis the United States. These are examples of the types of questions that the model developed here is suited to address.

The rest of the paper is organized as follows. Section 2 lays out the model. The equilibrium is characterized in Section 3, and the classical aggregation result of Houthakker (1955-1956) is extended to the dynamic equilibrium search setup in Section 4. This section also shows how, when aggregate inputs are correctly measured, the level of TFP depends on all the characteristics of the labor market summarized by the job-destruction decisions. Section 5 introduces four policies: employment subsidies, hiring subsidies, firing taxes and unemployment benefits, and studies their effects on TFP. Section 6 extends the basic model to the case of serially correlated shocks and state-dependent destruction rates, generalizes the main aggregation result, and elaborates on how the observed level of TFP is affected by the different ways of measuring aggregate inputs that can be found in the literature. Section 7 concludes. All propositions are proved in Appendix A. Additional extensions and technical results are collected in Appendix B.

2 The Model

The labor market is modeled as in Mortensen and Pissarides (1994).[2] Time is continuous, and the horizon infinite. There is a continuum of infinitely lived agents of two types: workers and firms. Both types are risk-neutral. The size of the labor force is normalized to unity while the number of firms will be determined endogenously by free entry. Workers derive utility from consumption and – in one of the specifications developed below – also suffer disutility from working. Each firm has a single job that can be either filled or vacant and searching. Similarly, workers can be either employed by a firm or unemployed and searching. No new offers arrive while an agent is in a relationship. I abstract from capital accumulation and assume an exogenous rental rate of capital, c. The aggregate stock of capital, K, will be determined by

[2] There are at least three reasons for carrying out the analysis in a search and matching framework. First, as will be discussed in Section 5, the labor-market policies considered will have testable implications not only for the level of TFP but also for the unemployment rate and the job-creation and destruction rates. Second, an explicit treatment of unemployment is relevant because – as will be shown in Section 6 – the unemployment rate will affect empirical measures of TFP for the ways of measuring aggregate inputs that can be found in the literature. And finally, this framework has been used extensively to analyze the effects of similar policies on many other aggregate labor-market outcomes (see Ljungqvist and Sargent [2000], Pissarides [2000] and references therein).

demand.[3]

Assume meeting frictions can be represented by a function $m(u,v)$ that determines the instantaneous number of meetings as a function of the numbers of searchers on each side of the market, namely, unemployed workers u and vacancies v. Suppose m exhibits constant returns to scale and is increasing in both arguments. Let $q(\theta)$ denote the (Poisson) rate with which a vacancy contacts an unemployed worker, where $\theta = v/u$. For some of the existence proofs it will also be useful to assume $\lim_{\theta\to 0} q(\theta) = \infty$.[4]

Each firm has access to a technology $f(x,n,k)$ that combines the hours supplied by the worker it employs, n, and capital, k, to produce a homogeneous consumption good. The match-specific level of technology is indexed by x. I assume that

$$f(x,n,k) = x \min(n,k) \tag{1}$$

and interpret k as the firm's capacity or scale of operation. So output is linear in hours but is bounded above by the stock of capital the firm is operating with. The convention is that the technology is such that all projects have the same scale of operation k. Every firm has to rent and put in place k units of capital to be able to engage in search while vacant and to produce while filled.[5] This captures the notion that hours are a fully flexible factor while capital is relatively fixed. Firms rent capital from a competitive market at flow cost c.

Match productivity is stochastic and indexed by the random variable x. The process that changes the productivity is Poisson with finite arrival rate λ. When a match of productivity x suffers a change, the new value x' is a draw from the fixed distribution $G(\cdot)$. So the productivity process is persistent (since $\lambda < \infty$) but – conditional on change – it is independent of the firm's

[3] This is the usual "small open economy" assumption. The model abstracts from saving and accumulation because the focus here is on isolating the effects of labor-market policies on the level of TFP. But even in the context of trying to explain income differences, Prescott (1998) and Parente and Prescott (2000) conclude that one cannot rely on policies that cause differences in saving rates, as they do not vary systematically with countries' incomes.

[4] Note that $q(\theta) = m(1/\theta, 1)$ and hence $q' < 0$. The probability a worker contacts a vacancy in a short time interval is $\theta q(\theta)$ and is increasing in θ.

[5] The idea is that in order to search, the firm must have borrowed some capital, say, to set up a plant, and plants come in a single size, k.

previous state.[6] The Poisson process and the productivity draws are *iid* across firms, and there is no aggregate uncertainty. The focus will be on steady-state outcomes.

In the next section I will show that there is a unique productivity level R_t such that active matches dissolve if productivity ever falls below that level and new matches form only if their initial productivity is at least R_t.[7] Let $H_t(x)$ denote the cross-sectional productivity distribution of active matches. That is, $H_t(x)$ is the fraction of matches producing at productivities x or lower at time t. The time path of $(1-u_t)H_t(x)$, the *number* of matches producing at productivities x or lower at time t, is given by[8]

$$\begin{aligned}\frac{d}{dt}\left[(1-u_t)H_t(x)\right] &= \lambda(1-u_t)\left[1-H_t(x)\right]\left[G(x)-G(R_t)\right]+\theta q(\theta)u_t\left[G(x)-G(R_t)\right]\\ &\quad -\lambda(1-u_t)H_t(x)G(R_t)-\lambda(1-u_t)H_t(x)\left[1-G(x)\right]\\ &\quad -\delta(1-u_t)H_t(x).\end{aligned}$$

The first term accounts for the matches with productivities above x that get innovations below x but above R_t. The newly formed matches that start off with productivities no larger than x are in the second term. The third term represents the matches in the interval $[R_t, x]$ that get shocks below R_t and are destroyed. The fourth term accounts for those matches in the same interval that "move up" by virtue of having drawn productivities larger than x. Let δ denote the parameter of an independent Poisson process that causes separations for reasons unrelated to the match-specific productivity shocks. Then the last term accounts for the matches in the

[6] This is the process used by Mortensen and Pissarides (1994). For reasons that will become clear shortly, Section 6 generalizes the model by specifying that when a match of productivity x suffers a change, the new value x' is a draw from the fixed distribution $G(\cdot|x)$. If $G(\cdot|x_1) < G(\cdot|x_0)$ when $x_0 < x_1$, then apart from being persistent, the idiosyncratic shocks are also positively correlated through time.

[7] Mortensen and Pissarides (1994) work with a bounded support and assume new matches start off with the highest productivity. I relax these assumptions and treat active and new matches symmetrically. In the model considered here, the initial productivity of a match is a nondegenerate random variable drawn from the same distribution as the innovations to active matches.

[8] The fact that active matches will form and continue only for productivities at least as large as R_t means that $H_t(R_t) = 0$. Thus the derivation focuses on $x \geq R_t$.

interval $[R_t, x]$ that are destroyed for exogenous reasons.[9] Imposing steady states,

$$H(x) = \left[\frac{\lambda}{\delta+\lambda} + \frac{\theta q(\theta) u}{(\delta+\lambda)(1-u)}\right][G(x) - G(R)].$$

In addition, the steady-state unemployment rate is

$$u = \frac{\delta + \lambda G(R)}{\delta + \lambda G(R) + \theta q(\theta)[1 - G(R)]}. \tag{2}$$

Using this expression, the steady-state cross-sectional productivity distribution becomes

$$H(x) = \frac{G(x) - G(R)}{1 - G(R)}. \tag{3}$$

Next, I consider the problems faced by a worker and a firm. The value of unemployment and employment to a worker are denoted U and $W(x)$, respectively, and solve

$$rU = b + \theta q(\theta) \int \max[W(z) - U, 0]\, dG(z) \tag{4}$$

$$rW(x) = w(x) + \lambda \int \max[W(z) - U, 0]\, dG(z) - (\delta + \lambda)[W(x) - U], \tag{5}$$

where r is the discount factor, $b \geq 0$ denotes a worker's flow income while unemployed, and $w(x)$ is the wage earned by a worker employed in a match of productivity x. This formulation assumes the worker suffers no disutility from work (but see Appendix B).

Firms can be either vacant and searching or filled. The problem of a searching firm is summarized by

$$rV = -ck + q(\theta) \int \max[J(z) - V, 0]\, dG(z), \tag{6}$$

where V is the asset value of a vacancy and $J(x)$ the asset value of a filled job. Letting $\pi(x)$ denote flow profit, $J(x)$ satisfies

$$rJ(x) = \pi(x) + \lambda \int \max[J(z) - V, 0]\, dG(z) - (\delta + \lambda)[J(x) - V], \tag{7}$$

where $\pi(x) = x \min(n, k) - w(x) - ck - \phi n - C(x, \phi) k$. Instantaneous profit is the residual output after the wage $w(x)$ and all other costs of production have been paid out. There are

[9] Section 6 extends the analysis to the case in which the rate δ is a decreasing function of the idiosyncratic productivity parameter x. The reasons why this extension may be worth exploring are discussed below.

three such costs in this formulation: the rental on capital, ck; a variable cost, ϕn, that can be managed by varying hours; and a fixed cost C per unit of capital.[10] The variable cost ϕn and the fixed cost Ck are introduced to allow for the possibility of "labor hoarding" and underutilization of capital, two pervasive features of the data. In the presence of these costs, for some parametrizations it will be possible that at low productivity realizations the firm chooses to keep the worker employed despite requiring that she supplies zero hours. In Section 4 I will show that this type of labor hoarding has interesting aggregate implications when it occurs in equilibrium.

To fix ideas, one can think of ϕ as the cost of electricity, for instance, with electricity usage being proportional to hours worked. Alternatively, in Appendix B I show how to modify (5) and $\pi(x)$ to get an equivalent formulation in which ϕn is the worker's disutility from supplying n hours to her employer. Either way, the key observation is that $\phi > 0$ is necessary for the model to display the type of labor hoarding described above. Intuitively, if the marginal product of labor, x, is lower than its marginal cost, ϕ (expressed either in terms of resources to the firm under the first interpretation or disutility to the worker under the second), then efficiency will require the match to set $n = 0$: the worker should not supply hours to the firm. Under these circumstances, whether the firm and the worker should choose to preserve the match is a different matter. One possibility is that the instances in which the firm-worker pair sets $n = 0$ are also the cases in which the pair chooses to dissolve the match. Or alternatively, one could imagine the pair may choose $n = 0$ but decide to preserve the match anticipating that the low realization of the idiosyncratic shock may be reversed soon. In the first scenario there is no hoarding. In the latter there is. The fixed cost C introduced in this section makes it possible for the second scenario to arise in the equilibrium with endogenous separations.

Notice that if $C = \phi = 0$ then the model reduces to the standard setup of Mortensen and Pissarides (1994). For this special case, all the results on how labor market policies affect the

[10] The cost C is fixed in the sense that the firm can only avoid it by shutting down, but – as the notation indicates – it may depend on the realization of the idiosyncratic shock and possibly also on the parameter ϕ. More on this below.

level of TFP still go through even though the equilibrium does not exhibit hoarding. In this sense, neither of these costs is essential. However, in the presence of the variable cost ϕ and the fixed cost C the model is in addition also able to deliver equilibrium hoarding. And as it turns out, this has interesting implications for how the aggregate production function looks.

Since the fixed cost C is perhaps the only nonstandard element of the model, several remarks are in order. First, at a technical level, incorporating a fixed-cost specification that is decreasing in the shock x is introduced in this section as a simple device to avoid a "flat spot" in flow profit which would otherwise carry over to the value functions.[11] Second, in Section 6 I drop the fixed cost and redo the analysis in a version of the model where the draws of the idiosyncratic shock are correlated over time. In another extension explored in Section 6, I again drop the fixed cost and show that the key insights also go through in a version of the model where the destruction rate δ is decreasing in the productivity shock. The point I want to stress here is that these alternative specifications seem natural (perhaps even more so than the usual benchmark in which the shocks hitting the match are independent of its state) and are able to deliver hoarding without the fixed cost C.[12] Given all this and for expositional purposes, I will – for the time being – use a particularly convenient specification for the fixed cost, namely, $C(x, \phi) = \max(\phi - x, 0)$.[13]

3 Equilibrium

I follow the bulk of the search literature by letting $\beta \in [0, 1)$ and assuming the instantaneous wage $w(x)$ and labor supply n solve

$$\max \ [W(x) - U]^{\beta} [J(x) - V]^{1-\beta}$$

[11] See Appendix B, and in particular the discussion around equation (46) for more on this.

[12] Intuitively, flow profit has a "flat spot" in both these alternative formulations, but the serially correlated shocks ensure that the value functions do not inherit this flat spot.

[13] This formulation is convenient because it will imply that the flow profit is affine and strictly increasing in x. Since – even accepting the presence of a fixed cost – this particular formulation may seem a bit contrived, in Appendix B I redo the whole analysis with a more general specification for the fixed cost and show that $\frac{\partial C(x,\phi)}{\partial x} < 0$ is all that is really needed. But again, see Section 6 for alternative specifications that do not rely on firms having to bear *any* fixed cost C.

at all times. The optimal choice of hours is

$$n(x) = \begin{cases} k & \text{if } \phi < x \\ 0 & \text{if } x \leq \phi, \end{cases} \tag{8}$$

and substituting it into the flow profit function gives $\pi(x) = (x - \phi - c)k - w(x)$ for all x. The first-order condition for the instantaneous wage $w(x)$ is

$$(1-\beta)[W(x) - U] = \beta[J(x) - V]. \tag{9}$$

Letting $S(x) = J(x) + W(x) - U - V$ denote the surplus from a match, notice that (9) implies $J(x) = (1-\beta)S(x)$ and $W(x) - U = \beta S(x)$. These together with (4), (5) and (7) imply

$$(r + \delta + \lambda) S(x) = (x - \phi - c)k - rU + \lambda \int \max[S(z), 0]\, dG(z),$$

where

$$rU = b + \frac{\beta}{1-\beta} kc\theta. \tag{10}$$

In deriving (10) I have already imposed that free entry of firms will make $rV = 0$ in equilibrium. Since $S'(x) = \frac{k}{r+\delta+\lambda} > 0$, there exists a unique R such that $S(x) > 0$ iff $x > R$. Hence a firm-worker pair destroys an existing match and chooses not to form a new match if it draws a productivity $x < R$.[14] Using this reservation strategy, the surplus can be written as

$$(r + \delta + \lambda) S(x) = (x - \phi - c)k - rU + \lambda \int_R S(z)\, dG(z). \tag{11}$$

For completeness, (9) and the value functions can be combined to obtain expressions for instantaneous wages and profit:

$$w(x) = \beta(x - \phi - c)k + (1-\beta) rU \tag{12}$$

$$\pi(x) = (1-\beta)[(x - \phi - c)k - rU]. \tag{13}$$

Intuitively, the wage is a weighted average of output (net of the rental on capital and the variable and fixed costs) and the worker's reservation wage.

[14] Notice that match formation and destruction are privately efficient. Moreover, they are also consensual in the sense that by (9), $J(x) > 0$ iff $W(x) - U > 0$; so the firm wants to destroy the match iff the worker wants to quit.

Next, I characterize the job-creation and destruction decisions as summarized by θ and R, respectively. Evaluating (11) at $x = R$,

$$\lambda \int_R S(z)\, dG(z) = rU - (R - \phi - c)\, k.$$

Notice that since the expected capital gain on the left-hand side is positive, at $x = R$ net output is smaller than the worker's reservation wage. Thus (12) and (13) imply that $w(R) < rU$ and $\pi(R) < 0$: workers and firms sometimes tolerate instantaneous payoffs below those they could get by separating, in anticipation of future productivity improvements.[15] Substituting this simpler expression for the expected capital gain term into (11) gives

$$S(x) = \frac{x - R}{r + \delta + \lambda} k. \tag{14}$$

Evaluating (11) at $x = R$ and using (14) to substitute $S(\cdot)$ yields what is usually referred to as the job-destruction condition:

$$R - \phi - c - \left(\frac{b}{k} + \frac{\beta}{1-\beta} c\theta\right) + \frac{\lambda}{r + \delta + \lambda} \int_R (x - R)\, dG(x) = 0. \tag{15}$$

As is standard, the destruction decision is independent of scale if b is. The natural interpretation of b is that it is unemployment insurance income. Along these lines, if one lets $b = \tau_b E_G[w(x) | x \geq R]$, where $\tau_b \in [0, 1)$ is the replacement rate, then $b = \hat{b}k$, with

$$\hat{b} = \frac{\tau_b \beta [\tilde{x}(R) - \phi - c + c\theta]}{1 - (1-\beta)\tau_b},$$

and $\tilde{x}(R) \equiv E_G[x | x \geq R] = [1 - G(R)]^{-1} \int_R x dG(x)$. Under this specification, b is linear in k so (15) is independent of k and becomes

$$R - \tfrac{\tau_b \beta \tilde{x}(R)}{1-(1-\beta)\tau_b} - \tfrac{(1-\tau_b)(\phi+c)}{1-(1-\beta)\tau_b} - \tfrac{\beta c\theta}{(1-\beta)[1-(1-\beta)\tau_b]} + \tfrac{\lambda}{r+\delta+\lambda} \int_R (x - R)\, dG(x) = 0.$$

In what follows I will always abstract from scale effects caused by unemployment income b by assuming it is a fraction of the average going wage.

[15] This feature of the model is a consequence of the costly and time-consuming meeting process, as noted by Mortensen and Pissarides (1994).

Substituting the equilibrium condition $rV = 0$ in (6) implies that

$$(1-\beta)\int_R S(x)\, dG(x) = \frac{ck}{q(\theta)}.$$

That is, the expected profit from a filled job equals the expected hiring cost in an equilibrium with free entry. Using (14) to substitute $S(\cdot)$ out of this expression yields what is often referred to as the job-creation condition:

$$\frac{1-\beta}{r+\delta+\lambda}\int_R (x-R)\, dG(x) = \frac{c}{q(\theta)}. \tag{16}$$

The job-creation and destruction conditions jointly determine R and θ, and under the maintained assumptions they are independent of scale, k.[16] For given c and ϕ, an equilibrium is a vector $[\theta, R, H, U, w, u, K]$ such that (θ, R) jointly solve (15) and (16); given (θ, R), H satisfies (3); U is given by (10); w is given by (12); and u is given by (2). In addition, the market for capital services should clear, so the aggregate supply of capital K must satisfy $K = [1-(1-\theta)u]\,k$, where the right-hand side is the total demand for capital (coming from both matched and unmatched firms). Note that in parametrizations that result in $R < \phi$, the capital and workers in matches with realizations in $[R, \phi)$ remain employed but are not engaged in production. The firms in these states have excess capacity and hoard labor. The following section provides a sharper characterization of aggregate outcomes for a particular distribution of idiosyncratic shocks.

4 Aggregation

Let K_e denote the capital in place at all the firms with filled jobs; that is, $K_e = (1-u)\,k$ or equivalently,

$$K_e = \frac{1-u}{1-(1-\theta)\,u}K. \tag{17}$$

Aggregate output, Y, and the total number of hours worked, N, are given by

$$Y = (1-u)\int_\mu f[x, n(x), k]\, dH(x)$$

[16]See Lemma 1 in Appendix B for conditions under which the pair (θ, R) that solves (15) and (16) exists and is unique.

and $N = (1-u)\int_{\mu} n(x)\, dH(x)$, respectively, with $\mu \equiv \max(R, \phi)$. Using (1) and (8),

$$Y(K_e, \mu) = [1 - H(\mu)] K_e E_H(x | x \geq \mu) \tag{18}$$

$$N = [1 - H(\mu)] K_e, \tag{19}$$

where $E_H(x|x \geq \mu) = [1 - H(\mu)]^{-1} \int_{\mu} x dH(x)$. Intuitively, since every firm-worker pair is setting hours either to zero or to full capacity k, the aggregate number of hours worked is just equal to the fraction of firm-worker pairs who engage in production times the total capital stock in filled jobs. Similarly, aggregate output equals the number of *active* units of capital, $[1 - H(\mu)] K_e$, weighted by their average productivity.[17] Following Houthakker (1955-1956), one could imagine solving (19) for the aggregate "labor demand" by active firms, $\mu(K_e, N)$, and then substituting it in (18) to obtain $Y[K_e, \mu(K_e, N)]$. Hereafter, I use $F(K_e, N)$ to denote $Y[K_e, \mu(K_e, N)]$ to simplify notation and stress the fact that this is the economy's "aggregate production function." Even for an arbitrary H, the aggregate production function exhibits constant returns to scale. To see this, notice that $\mu(K_e, N)$ is homogeneous of degree zero and hence (18) indicates that for any $\zeta > 0$, $F(\zeta K_e, \zeta N) = \zeta F(K_e, N)$.[18]

Now suppose idiosyncratic shocks are draws from a Pareto distribution with parameters ε and α, namely,

$$G(x) = \begin{cases} 0 & \text{if } x < \varepsilon \\ 1 - \left(\frac{\varepsilon}{x}\right)^{\alpha} & \text{if } \varepsilon \leq x, \end{cases} \tag{20}$$

[17] As mentioned previously, Mortensen and Pissarides (1994) assume that G has support $[0, 1]$ and that all new matches start off with productivity 1. So, with $\delta = 0$, aggregate output in their model evolves according to

$$\dot{Y} = \theta q(\theta) uk - \lambda Y + \lambda (1-u) k \int_{\mu}^{1} x dG(x).$$

Replacing $(1-u)k$ with K_e, steady state output is $Y = \frac{\theta q(\theta) uk}{\lambda} + [1 - H(\mu)] K_e E_H(x|x \geq \mu)$, which looks like (18) except for the first term. Assuming that the initial productivity of a new match is a random draw from G – just as the innovations to the productivity of ongoing matches – allows for a density G with unbounded support. In addition, this alternative assumption smoothes aggregate output by getting rid of the "spike" $\theta q(\theta) uk\lambda^{-1}$.

[18] Also, from (18) one sees that $F_2(K_e, N) = -\mu_2(K_e, N) K_e \mu dH(\mu)$, and from (19) that $-\mu_2(K_e, N) K_e dH(\mu) = 1$. Thus $F_2(K_e, N) = \mu$. So the marginal product of labor in the aggregate production function is equal to the marginal product of the least efficient unit of labor employed in production. I owe this argument to Erzo G. J. Luttmer. At this point it may be useful to stress that in this context, by an "aggregate production function" I mean *an equilibrium relationship between measured aggregate inputs and output*. The fact that this is an equilibrium relationship implies that in general, a change in parameters (e.g., unemployment benefits) will typically affect N, K_e, $\mu(.,.)$ and $Y(.,.)$.

where $\varepsilon > 0$ and $\alpha > 2$.[19] Then, provided $R \geq \varepsilon$, $1 - G(R) = \left(\frac{\varepsilon}{R}\right)^{\alpha}$ and hence $G(x) - G(R) = (\varepsilon/R)^{\alpha}\left[1 - (R/x)^{\alpha}\right]$ for any $x \geq R$. Substituting these expressions in (3) one sees that the steady state productivity distribution of active matches is

$$H(x) = \begin{cases} 0 & \text{if } x < R \\ 1 - \left(\frac{R}{x}\right)^{\alpha} & \text{if } R \leq x. \end{cases} \tag{21}$$

This is a Pareto distribution with parameters R and α. Using (21), $1 - H(\mu) = \left(\frac{R}{\mu}\right)^{\alpha}$ and $E_H(x|x \geq \mu) = \frac{\alpha}{\alpha-1}\mu$, so the aggregates (18) and (19) specialize to $Y(K_e, \mu) = \frac{\alpha}{\alpha-1}R^{\alpha}\mu^{1-\alpha}K_e$ and $N = (R/\mu)^{\alpha} K_e$. Inverting the latter to get the "aggregate labor demand" $\mu = (K_e/N)^{1/\alpha} R$ and substituting it in the former yields

$$F(K_e, N) = AK_e^{\gamma}N^{1-\gamma}, \tag{22}$$

where

$$A = \frac{R}{1-\gamma} \tag{23}$$

and $\gamma \equiv 1/\alpha$. This extends the classic aggregation result of Houthakker (1955-1956).[20] The factor A is what macroeconomists normally refer to as TFP. Its level depends on α, a parameter of the primitive distribution of productivity shocks, as well as on all the characteristics of the labor market as summarized by the destruction decision R.

Notice that F expresses output as a function of the aggregate number of *hours worked*, N, and the total amount of capital hired by firms with filled jobs, K_e. One can also express output as a function of the aggregate capital stock, K, simply by substituting (17) in (22) to get

[19] This distribution has mean $\bar{x} = \frac{\alpha}{\alpha-1}\varepsilon$ and variance equal to $\frac{\bar{x}}{(\alpha-2)(\alpha-1)}\varepsilon$. Assuming $\alpha > 2$ ensures both are well-behaved.

[20] Houthakker performed the aggregation over production units that employ two variable factors and face capacity constraints due to a fixed (unmodelled) factor. Here I have assumed each production unit employs a single variable factor (labor) as well as capital. Capital is chosen before engaging in search and then remains fixed, hence playing the role of the fixed factor constraining output at the time employment and production decisions are made. This formulation delivers an aggregate production function with constant returns to scale. In contrast, the setup used by Houthakker generates a function of the variable inputs only and it exhibits diminshing returns to scale. Another relevant difference is that the shift parameter in Houthakker's production function is solely a function of the parameters in the primitive productivity distribution. But here, decisions can shift the aggregate production function. In a different context, Jones (2004) also obtains a Cobb-Douglas aggregate when the underlying heterogeneity is Pareto.

$\hat{F}(K,N) = \hat{A}K^{\gamma}N^{1-\gamma}$, where $\hat{A} \equiv \left[\frac{1-u}{1-(1-\theta)u}\right]^{\gamma} A$. (More on this in Section 6.2.) The aggregate production function is Cobb-Douglas despite fixed proportions in the micro-level technologies. This results whenever utilization is imperfectly measured, namely, when not all of the capital stock included as an argument in the aggregate production function is actually being used in production.[21] Since having firm-worker pairs that sometimes choose to be inactive affects the shape of the aggregates, I next verify that it is indeed possible for the equilibrium to exhibit this property.

With G given by (20), (15) and (16) specialize to

$$\left[1 - \frac{\tau_b\beta\alpha}{(\alpha-1)[1-(1-\beta)\tau_b]}\right] R - \frac{(1-\tau_b)(\phi+c)}{1-(1-\beta)\tau_b} - \frac{\beta c\theta}{1-\beta} + \frac{\lambda\varepsilon^{\alpha}R^{1-\alpha}}{(\alpha-1)(r+\delta+\lambda)} = 0 \tag{24}$$

$$\frac{\varepsilon^{\alpha}R^{1-\alpha}}{\alpha-1} - \frac{(r+\delta+\lambda)\,c}{q(\theta)(1-\beta)} = 0. \tag{25}$$

Lemma 2 in Appendix B provides conditions that ensure a unique equilibrium exists for this formulation. Since the purpose of the remainder of this section is merely to show that hoarding *may* arise in equilibrium, I set $\tau_b = 0$ to ease the algebra.[22] Differentiating (24) and (25),

$$\frac{\partial R}{\partial \phi} = \frac{(r+\delta+\lambda)\eta(\theta)}{\beta\theta q(\theta)[1-G(R)] + (r+\delta+\lambda)\eta(\theta)\left\{1 - \frac{\lambda[1-G(R)]}{r+\delta+\lambda}\right\}} > 0 \tag{26}$$

and

$$\frac{\partial \theta}{\partial \phi} = \frac{-(1-\beta)\,\theta q(\theta)\,[1-G(R)]}{(r+\delta+\lambda)\,\eta(\theta)\,c}\frac{\partial R}{\partial \phi} < 0,$$

where $1 - G(R) = (\varepsilon/R)^{\alpha}$ and $\eta(\theta) \equiv -\theta q'(\theta)/q(\theta)$. An increase in ϕ has no direct effect on the job-creation condition, and it shifts the job-destruction condition up in θ-R space. This increases the equilibrium value of R and decreases the equilibrium value of θ. Combining (24)

[21] To see this, notice that if there is no hoarding in equilibrium (i.e., if $\mu = R$) then $N = K_e$ and $F(K_e, N) = AK_e$. Similarly, if there is hoarding but utilization is perfectly measured, then aggregate output is again linear in the relevant capital stock. Explicitly, let K_p denote the capital stock being used in production, that is, $K_p = [1 - H(\mu)]\,K_e$. Then it follows from (18) that $Y = A''K_p$, with $A'' \equiv E_H(x|x \geq \mu)$. So anything less than perfect measurement of capital utilization together with some degree of hoarding cause the aggregate to look Cobb-Douglas in capital and hours despite fixed proportions in the micro production functions.

[22] Just as for the existence and uniqueness results in Lemma 1 and Lemma 2 in Appendix B, the results that follow will also hold for replacement rates τ_b that are not too big. The algebra gets cumbersome with $\tau_b > 0$ because under this specification unemployment income b is a function of the equilibrium distribution of wages earned.

and (25), one sees that the sign of $\phi - R$ is the sign of $\lambda/q(\theta) - [1-(1-\theta)\beta]$. So at low productivity realizations, the firm is more likely to hoard labor than to break the match when λ is large (and hence the option value of keeping a match is large) and when q is small (and hence the expected cost of hiring a new worker is high). Market tightness θ enters the expression with an ambiguous sign because on the one hand a large θ makes hoarding more likely by increasing the expected recruiting cost; but on the other, through its effect on the worker's reservation wage, it also increases the value of her threat point in the wage bargain, which makes keeping an unproductive worker employed more costly and hoarding less likely. In fact, the latter effect disappears if the worker has no power in the wage bargain (i.e., if $\beta = 0$). Next, I derive a condition on parameters that is sufficient for $R < \phi$ in equilibrium.

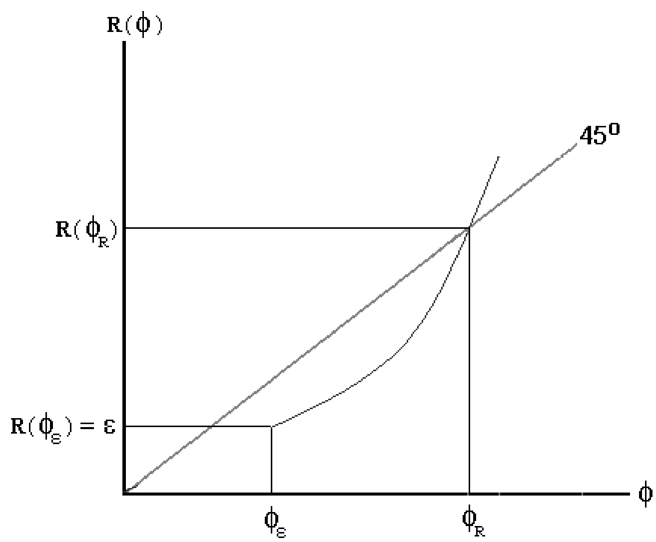

Figure 1: Destruction decision as a function of the variable cost.

Let θ_ε^* be defined by $q(\theta_\varepsilon^*) = \frac{(\alpha-1)(r+\delta+\lambda)c}{(1-\beta)\varepsilon}$ and $\phi_\varepsilon = \left[1 + \frac{\lambda}{(\alpha-1)(r+\delta+\lambda)}\right]\varepsilon - \left(1 + \frac{\beta}{1-\beta}\theta_\varepsilon^*\right)c$. Then if $\phi = \phi_\varepsilon$, the versions of (24) and (25) with $\tau_b = 0$ are solved by $\theta(\phi_\varepsilon) = \theta_\varepsilon^*$ and $R(\phi_\varepsilon) = \varepsilon$. Notice that if $R(\phi_\varepsilon) = \varepsilon < \phi_\varepsilon$, then there is a nondegenerate interval $[\phi_\varepsilon, \phi_R)$ such that $R(\phi) < \phi$ iff $\phi \in [\phi_\varepsilon, \phi_R)$. An example of the function $R(\phi)$ is illustrated in Figure 1.

So a sufficient condition for the equilibrium to exhibit hoarding for at least some range of the parameter ϕ is that $\phi_{\varepsilon} - \varepsilon > 0$, or equivalently, that $T(\lambda, \zeta) > 0$, where

$$T(\lambda, \zeta) \equiv \frac{\lambda \varepsilon}{(\alpha - 1)(r + \delta + \lambda)} - \left(1 + \frac{\beta}{1 - \beta} \theta_{\varepsilon}^{*}\right) c.$$

The parameter ζ summarizes the efficiency of matching, with the property that $\partial m(u, v) / \partial \zeta > 0$ and hence that $\partial q(\theta) / \partial \zeta > 0$ for all θ. Figure 2 plots the boundary $T(\lambda, \zeta) = 0$ in λ-ζ space.

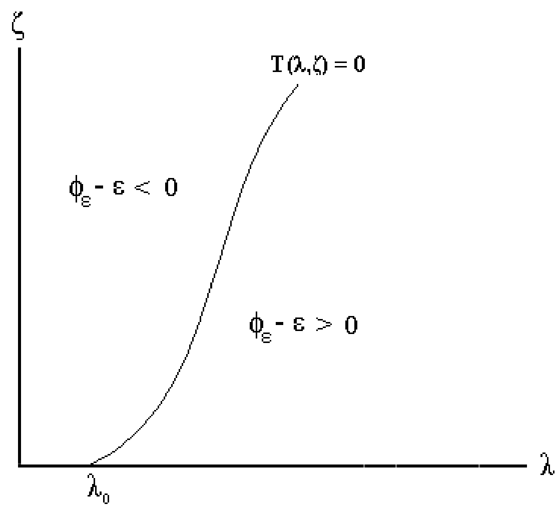

Figure 2: Range of parameters for which there is hoarding.

The condition $\phi_{\varepsilon} - \varepsilon > 0$ is satisfied for the values of the parameters λ and ζ that lie below the boundary.[23] Intuitively, the parameter restriction that makes hoarding possible holds for relatively large λ (i.e., when bad shocks are very transitory) and relatively low ζ (i.e., when the search process needed to replace the worker is very costly). Having characterized the relevant properties of the equilibrium, the following section studies the effects of labor-market policies on the level of TFP.

[23] Note that θ_{ε}^{*} goes to zero as ζ goes to zero. So $T(\lambda, 0) = 0$ iff $\lambda = \lambda_0$, where $\lambda_0 \equiv \frac{c(\alpha-1)(r+\delta)}{\varepsilon - c(a-1)}$ is the point at which the boundary intercepts the horizontal axis in Figure 2. Formally, this boundary is upward-sloping because $\frac{\partial T}{\partial \zeta} = -\frac{\beta \theta_{\varepsilon}^{*}}{1-\beta} \frac{\partial \theta_{\varepsilon}^{*}}{\partial \zeta} < 0$ and $\frac{\partial T}{\partial \lambda} = \frac{\varepsilon(r+\delta)}{(\alpha-1)(r+\delta+\lambda)^2} - \frac{bc^2(\alpha-1)}{(1-\beta)\varepsilon(1-\beta)q'\left(\theta_{\varepsilon}^{*}\right)} > 0$. The equilibrium may or may not exhibit hoarding for parametrizations that lie above the boundary.

5 Labor-Market Policies and the Level of TFP

This section considers the effects of four policies: employment subsidies, hiring subsidies, firing taxes and unemployment benefits. I follow Pissarides (2000) and model the subsidies as transfers from the government to the firm and the firing tax as a payment from the firm to the government.[24] The value function $W(x)$ is still given by (5), while (4), (6) and (7) now generalize to

$$\begin{aligned} rU &= b + \theta q(\theta) \int \max\left[W_o(z) - U, 0\right] dG(z) \\ rV &= -ck + q(\theta) \int \max\left[J_o(z) + \tau_h k - V, 0\right] dG(z) \\ rJ(x) &= \pi(x) + \tau_e k + \lambda \int \max\left[J(z) - V + \tau_f k, 0\right] dG(z) - (\delta + \lambda)\left[J(x) - V + \tau_f k\right]. \end{aligned}$$

The policy variables are τ_h (hiring subsidy), τ_e (employment subsidy), τ_f (firing tax) and b (unemployment benefit). Note that all payments are assumed to be proportional to the firm's size, as measured by k.[25] There are two reasons why the bargaining situation faced by a firm and a worker when they first meet and are still considering whether to form a match is different from the one they face every instant after having agreed to form the match. The first is that in the initial bargain there is a one-time hiring subsidy at stake. The second is that at that point the firm is not yet "locked in" by the firing tax. I use $w_o(x)$ to denote the wage that solves the initial bargain and $w(x)$ to denote the subsequent wage. Accordingly, $J_o(x)$ and $W_o(x)$ are the value functions for the firm and worker in the instant they form a new match and satisfy $W_o(x) - W(x) = J(x) - J_o(x) = w_o(x) - w(x)$.

The wages $w_o(x)$ and $w(x)$ are characterized by

$$\beta\left[J_o(x) + \tau_h k\right] = (1-\beta)\left[W_o(x) - U\right] \quad \text{and} \quad \beta\left[J(x) + \tau_f k\right] = (1-\beta)\left[W(x) - U\right],$$

[24] I assume that upon separation the firm must pay the firing tax to the government because in the present setup, firing taxes would be completely neutral under the alternative scheme where the firm compensates the fired worker directly.

[25] This assumption, borrowed from Pissarides (2000), is useful here because it ensures that policies introduce no scale effects into the job-creation and destruction decisions. Also, to keep the analysis simple, the government's financing constraints will be ignored. A natural extension would be requiring the government to run a balanced budget. A simple example of a scheme which is self-financing in the steady state is $\tau_f = \tau_h$ and $\tau_b = \tau_e = 0$.

where the equilibrium condition $V = 0$ has already been imposed. Letting $S_o(x) = J_o(x) + W_o(x) + \tau_h k - U$ and $S(x) = J(x) + W(x) + \tau_f k - U$ be the initial and the subsequent surpluses, respectively, the first-order conditions imply that $W_o(x) - U = \beta S_o(x)$, $W(x) - U = \beta S(x)$, $J_o(x) + \tau_h k = (1-\beta) S_o(x)$ and $J(x) + \tau_f k = (1-\beta) S(x)$. Combining these with the value functions gives

$$(r+\delta+\lambda) S(x) = (x - \phi - c) k + \tau_e k + r\tau_f k - rU + \lambda \int \max[S(z), 0] \, dG(z),$$

with rU as in (10). Since $S'(x) > 0$, there is a unique R such that $S(x) \geq 0$ iff $x \geq R$. Using this reservation property, the surplus of an ongoing match can be written as

$$(r+\delta+\lambda) S(x) = (x - \phi - c) k + \tau_e k + r\tau_f k - rU + \lambda \int_R S(z) \, dG(z), \tag{27}$$

a natural generalization of (11). One can work with the value functions and the first-order conditions of the Nash problem to derive expressions for wages and profit.[26] Evaluating (27) at $x = R$ implies

$$\lambda \int_R S(z) \, dG(z) = rU - [(R - \phi - c) k + \tau_e k + r\tau_f k],$$

and substituting this back into (27) yields (14). Using (14) to substitute $S(z)$ out of (27), evaluating at $x = R$ and using (10) produces the job-destruction condition:

$$R - \phi - c + \tau_e + r\tau_f - \left(\tau_b + \frac{\beta}{1-\beta} c\theta\right) + \frac{\lambda}{r+\delta+\lambda} \int_R (x - R) \, dG(x) = 0.$$

Increases in the employment subsidy and the firing tax reduce R for given θ. In other words, an increase in τ_e or τ_f shifts the job-destruction condition down in θ-R space. Conversely, an increase in τ_b raises the worker's outside option and hence increases R for given θ.

By free entry, $rV = 0$, and hence

$$(1-\beta) \int \max[S_o(x), 0] \, dG(x) = \frac{ck}{q(\theta)}. \tag{28}$$

[26] The wages and profit in ongoing matches are given by $w(x) = \beta(x - \phi - c + \tau_e + r\tau_f) k + (1-\beta) rU$ and $\pi(x) = (1-\beta)[(x - \phi - c) k - rU] - \beta(\tau_e + r\tau_f) k$, while those agreed upon in an initial match are $w_o(x) = w(x) + \beta(r+\delta+\lambda)(\tau_h - \tau_f) k$ and $\pi_o(x) = \pi(x) - \beta(r+\delta+\lambda)(\tau_h - \tau_f) k$. All matches still set hours according to (8).

Note that $S_o(x) = S(x) + (\tau_h - \tau_f)k$, so in the presence of firing and hiring policies, the reservation strategy used for match formation, say R_o, may differ from R, the one used for match dissolution. However, there are several specifications for the hiring subsidy and the destruction tax under which $R_o = R$, and the analysis will focus on this subset of policies hereafter.[27] Then (28) becomes

$$(1-\beta)\int_R S(x)\,dG(x) = \frac{ck}{q(\theta)},$$

which together with (14) yields the job-creation condition:

$$\frac{1-\beta}{r+\delta+\lambda}\int_R (x-R)\,dG(x) + (1-\beta)\left[1-G(R)\right](\tau_h-\tau_f) = \frac{c}{q(\theta)}.$$

Finally, assuming G is as in (20), the job-destruction and creation conditions specialize to

$$R - \phi - c + \tau_e + r\tau_f - \left(\tau_b + \tfrac{\beta}{1-\beta}c\theta\right) + \tfrac{\lambda\varepsilon^\alpha R^{1-\alpha}}{(\alpha-1)(r+\delta+\lambda)} = 0 \qquad (29)$$

$$\tfrac{\varepsilon^\alpha R^{1-\alpha}}{(\alpha-1)(r+\delta+\lambda)} + \left(\tfrac{\varepsilon}{R}\right)^\alpha(\tau_h-\tau_f) - \tfrac{c}{q(\theta)(1-\beta)} = 0. \qquad (30)$$

The main properties of the equilibrium are summarized as follows.

Proposition 1 *Suppose* $\varepsilon + (\alpha-1)(r+\delta+\lambda)(\tau_h-\tau_f) > 0$. *Let* θ^*_ϵ *be defined by* $q(\theta^*_\epsilon) = \frac{(\alpha-1)(r+\delta+\lambda)c}{(1-\beta)\left[\varepsilon+(\alpha-1)(r+\delta+\lambda)(\tau_h-\tau_f)\right]}$ *and* $\phi_\epsilon = \left[1+\frac{\lambda}{(\alpha-1)(r+\delta+\lambda)}\right]\varepsilon - \left(1+\frac{\beta}{1-\beta}\theta^*_\epsilon\right)c + \tau_h + r\tau_f - \tau_b$. *If* $\varepsilon + \alpha(r+\delta+\lambda)(\tau_h-\tau_f) > 0$, *then for any* $\phi > \phi_\varepsilon$ *(a) there exists a unique equilibrium, (b)* $R > \varepsilon$, *(c)* $\partial R/\partial\phi > 0$ *and (d)* $\partial\theta/\partial\phi < 0$. *If in addition* $\phi_\epsilon - \varepsilon > 0$, *then (e) there is a nondegenerate interval* $(\phi_\epsilon, \widetilde{\phi})$ *such that* $R(\phi) < \phi$ *for all* $\phi \in (\phi_\epsilon, \widetilde{\phi})$.

Aggregate output is still given by (22); the aggregate stock of capital in filled jobs, K_e, is still given by (17); and the aggregate number of hours worked, N, is still given by $N = (R/\mu)^\alpha K_e$.

[27] The simplest example of one such policy is the self-financing policy that sets $\tau_h = \tau_f$ for all values of the idiosyncratic productivity draw x. But more generally, a policy specifying $\tau_h = \tau_f$ for all realizations $x \le R$ and $\tau_h > \tau_f$ for all $x > R$ would also imply $R_o = R$. Note that a policy that merely specifies $\tau_h > \tau_f$ for all x will be abused by firms and workers in the sense that since $R_o < R$, a firm-worker pair that meets and draws $x \in (R_o, R]$ will want to form a match only for an instant to collect $(\tau_h - \tau_f)k$ and destroy it right away (since right after collecting from the government their break-up decision is given by R). Yet another set of policies delivering $R_o = R$ is having $\tau_f > \tau_h = 0$ but assuming τ_f is enforced from the very instant the firm-worker pair meet and start bargaining, which implies $w_o(x) = w(x)$ since in this case the firm is "locked in" by the firing tax upon meeting the worker.

In addition, if the measure of capital used to construct aggregate output is K_e, then the level of TFP is again given by (23). The following result, which holds under the assumptions stated in Proposition 1, summarizes the effects that labor market policies have on the level of TFP.

Proposition 2 *Employment subsidies and firing taxes reduce A. Hiring subsidies and unemployment benefits increase A.*

Since A is proportional to R, policies have the same qualitative effect on TFP as on the destruction rate. Proposition 2 is illustrated in Figure 3.

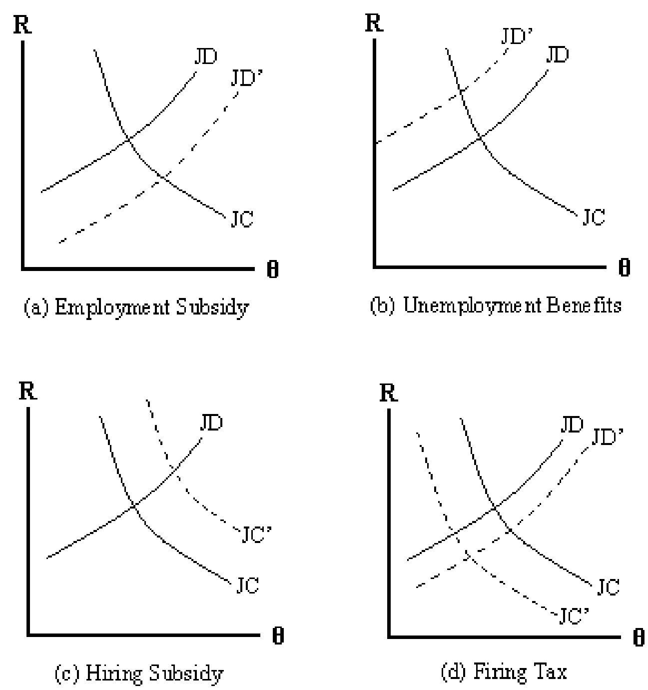

Figure 3: Equilibrium effects of various policies.

Employment subsidies make firms more tolerant of low productivity realizations and hence lower the average productivity of active firms. All else equal, an economy with relatively high subsidies to continued employment will exhibit a low job-destruction rate, a high job-creation rate, and hence low levels of unemployment and measured TFP. Firing taxes have a similar qualitative effect on job-destruction, but that mechanism is reinforced by a relatively low rate of job-creation (which reduces the reservation wage and hence makes firms even more tolerant of low productivity realizations). So firing restrictions will reduce measured TFP, as well as the job-creation and destruction rates. Hiring subsidies have no direct effect on the destruction decision, but they stimulate job-creation. This increases market tightness which in turn increases the workers' outside option and raises measured TFP, job-creation and destruction. Unemployment benefits also cause R to rise through an increase in the worker's reservation wage. Consequently, economies with relatively high unemployment benefits will tend to exhibit relatively high levels of TFP and unemployment.

6 Extensions

This section extends the basic model to the case of serially correlated shocks, generalizes the main aggregation result, and shows how the observed level of TFP is affected by the various ways of measuring aggregate inputs that can be found in the literature.

6.1 Correlated Shocks

Section 4 established that, with some mismeasurement and equilibrium hoarding, the standard search model of the labor market with a particular structure of shocks generates a relationship between aggregate inputs and output that looks exactly like the standard Cobb-Douglas relation typically used in growth accounting exercises. The fixed cost $C(x, \phi)$ was introduced in Section 2 as a simple device to avoid "flat spots" in the value functions, and this made it possible for the equilibrium to exhibit hoarding.[28] Here I show that by extending the model in a natural

[28] For $C(x, \phi) = 0$, $\pi(x) = [\max(x - \phi, 0) - c]\, k - w(x)$, so $\pi(x)$ is flat up to ϕ and then rises with slope k. It is easy to show that in this case $J(x)$ is also flat up to ϕ and then rises with slope $\frac{k}{r+\delta+\lambda}$. Note that since R is defined by $J(R) = 0$, this implies that generically the equilibrium with endogenous destruction will have $\phi < R$; i.e., there is no hoarding except for the knife-edge case in which R is indeterminate. Ruling out these types of flat spots in J allows for the possibility that $R < \phi$ in an equilibrium with endogenous destruction. See Appendix B for details.

way, one can drop the fixed cost without affecting the main results. To this end, I generalize the productivity process by allowing for serially correlated shocks: when a match of productivity x suffers a change, the new value x' is a draw from the fixed distribution $G(x'|x)$. Assuming $G(x|x_1) < G(x|x_0)$ if $x_0 < x_1$ allows idiosyncratic shocks to be positively correlated through time. For this case, the cross-section of productivities evolves according to

$$\begin{aligned}
\frac{d}{dt}\left[(1-u_t) H_t(x)\right] \quad = \quad & \lambda (1-u_t) \int_x^{\infty} \left[G(x|s) - G(R_t|s)\right] dH_t(s) \\
& + \theta q(\theta) u_t \int_{-\infty}^{\infty} \left[G(x|s) - G(R_t|s)\right] dH_t(s) \\
& - \lambda (1-u_t) \int_{-\infty}^{x} G(R_t|s)\, dH_t(s) \\
& - \lambda (1-u_t) \int_{-\infty}^{x} \left[1 - G(x|s)\right] dH_t(s) - \delta (1-u_t) H_t(x).
\end{aligned}$$

The first term accounts for the matches with productivities above x that get innovations below x but above R_t. The newly formed matches that start off with productivities no larger than x are in the second term. Notice the assumption that upon contact, the worker and firm draw their productivity level from the density corresponding to the average productivity among active matches.[29] The third term is the number of matches in the interval $[R_t, x]$ that get shocks below R_t and are destroyed. The fourth term accounts for the number of matches in the same interval that "move up" by virtue of having drawn productivities larger than x. The last term accounts for matches in the interval $[R_t, x]$ that are destroyed for exogenous reasons. Imposing steady states and re-arranging,

$$H(x) = \left[\frac{\lambda}{\delta+\lambda} + \frac{\theta q(\theta) u}{(\delta+\lambda)(1-u)}\right] \int \left[G(x|s) - G(R|s)\right] dH(s).$$

[29] When shocks are *iid*, one can specify that new matches draw z from $G(z)$ just as active matches do when forced to update their shock. However, with correlated shocks active matches with state z draw the new shock z' from $G(z'|z)$. Since vacancies and unemployed workers have no productivity attached to them, I assume their initial draw z' is from the average density $\int G(z'|z)\, dH(z)$. As a way of motivating this, imagine – as do Mortensen and Pissarides (1994) – that firms must irreversibly adopt a "technology" to engage in production. The present specification then means that they draw their technology at random from all those active at the time the match is created. Jeffrey Campbell pointed out to me that Conlisk (1989) uses a similar assumption to determine the productivity of newly created plants in a model of technical change.

The steady-state unemployment rate is given by

$$u=\frac{\delta+\lambda\int G\left(R|s\right)dH\left(s\right)}{\delta+\lambda\int G\left(R|s\right)dH\left(s\right)+\theta q\left(\theta\right)\int\left[1-G\left(R|s\right)\right]dH\left(s\right)}. \tag{31}$$

Using this expression, the steady-state cross-sectional productivity distribution becomes

$$H\left(x\right)=\frac{\int\left[G\left(x|s\right)-G\left(R|s\right)\right]dH\left(s\right)}{\int\left[1-G\left(R|s\right)\right]dH\left(s\right)}, \tag{32}$$

a natural generalization of (3).

The problem of a searching firm is now summarized by

$$rV=-ck+q\left(\theta\right)\int\int\max\left[J\left(z\right)-V,0\right]dG\left(z|x\right)dH\left(x\right). \tag{33}$$

And again, there will be entry of firms until all rents are exhausted, so $rV=0$ in equilibrium. The value of a filled job with productivity x is

$$rJ\left(x\right)=\pi\left(x\right)+\lambda\int\max\left[J\left(z\right)-V,0\right]dG\left(z|x\right)-\left(\delta+\lambda\right)\left[J\left(x\right)-V\right], \tag{34}$$

where $\pi\left(x\right)=x\min\left(n,k\right)-\phi n-ck-w\left(x\right)$. Flow profit $\pi\left(x\right)$ is the residual remaining after the wage $w\left(x\right)$ and all other costs of production have been paid out. There are only two such costs in this formulation: the rental rate, ck, and the variable cost, ϕn.[30] The choice of hours that solves the bargaining problem is still given by (8), and hence $\pi\left(x\right)=y\left(x\right)-w\left(x\right)$, where $y\left(x\right)\equiv\left[\max\left(x-\phi,0\right)-c\right]k$ is output net of the variable cost and the rental on capital.

The values of unemployment and employment to a worker are

$$rU = b+\theta q\left(\theta\right)\int\int\max\left[W\left(z\right)-U,0\right]dG\left(z|x\right)dH\left(x\right) \tag{35}$$

$$rW\left(x\right) = w\left(x\right)+\lambda\int\max\left[W\left(z\right)-U,0\right]dG\left(z|x\right)-\left(\delta+\lambda\right)\left[W\left(x\right)-U\right], \tag{36}$$

where $w\left(x\right)$ it is still characterized by (9). Letting $S\left(x\right)=J\left(x\right)+W\left(x\right)-U$ denote the surplus from a match, notice that (9) implies $J\left(x\right)=\left(1-\beta\right)S\left(x\right)$, $W\left(x\right)-U=\beta S\left(x\right)$, $\pi\left(x\right)=\left(1-\beta\right)\left[y\left(x\right)-rU\right]$ and

$$w\left(x\right)=\beta y\left(x\right)+\left(1-\beta\right)rU. \tag{37}$$

[30] Here I model ϕn as a cost borne by the firm, but Appendix B shows that this formulation is equivalent to one where ϕn is instead the disutility the worker experiences from working n hours.

Combining (9) with (34), (35) and (36) yields

$$(r+\delta+\lambda)\,S(x) = y(x) - rU + \lambda \int \max\left[S(z),0\right] dG(z|x),$$

where rU is given by (10). The fact that $S'(x) > 0$ implies that there exists a unique R such that $S(x) > 0$ iff $x > R$. Hence matches separate (or don't form) for productivity draws below R. Moreover, notice that $S(x)$ is strictly increasing for all x (even for $x < \phi$) despite the fact that $y(x)$ is flat for $x < \phi$. This is because $dG(\cdot|x)$ is stochastically increasing in x. Consequently, the equilibrium may have $R > \phi$ (no hoarding) for some parametrizations and $R < \phi$ (hoarding) for others. The logic of Figure 2 still applies in trying to determine which parametrizations may exhibit hoarding. Next, I turn to the issue of aggregation.

Suppose idiosyncratic shocks are draws from

$$G(x|s) = \begin{cases} 0 & \text{if } x < \varepsilon(s) \\ 1 - \left[\frac{\varepsilon(s)}{x}\right]^{\alpha} & \text{if } \varepsilon(s) \leq x, \end{cases}$$

where $\varepsilon(\cdot)$ is a continuously differentiable function and $\alpha > 2$. I introduce positively correlated shocks by assuming that $\varepsilon' > 0$ (if $\varepsilon' = 0$ the distribution reduces to (20), the special case of uncorrelated shocks). In addition, suppose there is an $\underline{\varepsilon} > 0$ such that $\varepsilon(\underline{\varepsilon}) = \underline{\varepsilon}$ and $\varepsilon(s) = 0$ if $s < \underline{\varepsilon}$, and that $\lim_{s\to\infty} \varepsilon(s) = 1 + \underline{\varepsilon} \equiv \overline{\varepsilon}$.[31]

Then for $R \geq \varepsilon(s)$, $1 - G(R|s) = \left[\frac{\varepsilon(s)}{R}\right]^{\alpha}$; hence, for any $x \geq R$, $G(x|s) - G(R|s) = \left[\varepsilon(s)/R\right]^{\alpha}\left[1 - (R/x)^{\alpha}\right]$. After substituting these expressions in (32) it becomes clear that the steady state productivity distribution of active matches is still given by (21). So for this case, the job-creation and destruction conditions are

$$\begin{aligned} \frac{\mu^{1-\alpha}R^{\alpha}}{\alpha-1} - \frac{\alpha[y(R)-rU]}{k}\left[\frac{R}{\varepsilon(R)}\right]^{\alpha}\int_R \frac{\varepsilon(x)^{\alpha}-\varepsilon(R)^{\alpha}}{x^{1+\alpha}}dx - \frac{(r+\delta+\lambda)c}{(1-\beta)q(\theta)}\left[\alpha\int_R \frac{\varepsilon(x)^{\alpha}}{x^{1+\alpha}}dx\right]^{-1} &= 0 \\ \left[1 - \frac{\lambda\alpha}{r+\delta+\lambda}\int_R \frac{\varepsilon(x)^{\alpha}-\varepsilon(R)^{\alpha}}{x^{1+\alpha}}dx\right]\frac{y(R)-rU}{k} + \frac{\lambda}{r+\delta+\lambda}\frac{\varepsilon(R)^{\alpha}\mu^{1-\alpha}}{\alpha-1} &= 0. \end{aligned}$$

An equilibrium is still a list $[R,\theta,H,U,w,u,K]$ such that R, θ and H jointly solve (32) and the job-creation and the job-destruction conditions, rU is given by (10), w is given by (37), and u

[31] An example of a function $\varepsilon(\cdot)$ satisfying all these conditions is $\varepsilon(s) = 1 + \underline{\varepsilon} - e^{\underline{\varepsilon}-s}$, for any $\underline{\varepsilon} > 0$.

satisfies (31). In addition, the market for capital should clear, so $[1-(1-\theta)u]k = K$. Sufficient conditions can be found so that the job-creation condition slopes down and the destruction condition up in θ-R space, implying a unique (θ, R) pair. A parameter restriction analogous to the one depicted in Figure 2 guaranteeing that there is a range of values for ϕ such that $R < \phi$ can still be derived.[32] Given the equilibrium pair (θ, R), the same procedure followed in Section 4 reveals that output aggregates to (22). Thus the key aggregation result obtained in the context of the model of Section 2 does not hinge on some of the particular modelling choices made there. In particular, allowing for correlated shocks can generate hoarding, and there is no need for the fixed cost of Section 2.

6.2 More on Aggregation

In this subsection I prove two additional aggregation results. First, I provide another specification of primitives that delivers the same aggregation result obtained for the models of Sections 2 and 6.1. Again, this specification does not rely on the fixed cost of Section 2, so in a way it reinforces the message of Section 6.1. The second aggregation result shows how to "reverse engineer" a distribution of idiosyncratic shocks that gives rise to an aggregate CES production function.

First consider the model of Section 2, but with $C = 0$ and separation rates that are decreasing in the productivity of the match; i.e., $0 < \delta(x) < \infty$ for all x, with $\delta' < 0$. The interpretation is that δ is a technological parameter: in any small time-interval, with probability $\delta(x)$ the idiosyncratic productivity of the match jumps to zero and stays at that level forever. I return to the case of uncorrelated productivity draws for the remainder of the section.

[32] Showing that equilibria with $R < \phi$ are possible for some parametrizations is now rather tedious, so the basic idea is only outlined here. Let $\phi_{\overline{\varepsilon}}$ be the value of ϕ such that $\theta^*_{\overline{\varepsilon}}$ and $R(\phi_{\overline{\varepsilon}}) = \overline{\varepsilon}$ solve the job-creation and destruction conditions. Then if $\phi_{\overline{\varepsilon}} - \overline{\varepsilon} > 0$, there will be an interval $(\phi_{\overline{\varepsilon}}, \widehat{\phi})$ such that $R(\phi) < \phi$ iff $\phi \in (\phi_{\overline{\varepsilon}}, \widehat{\phi})$. If, in addition, $\partial R(\phi)/\partial\phi > 0$, then $\phi_{\overline{\varepsilon}} - \overline{\varepsilon} > 0$ also implies $\overline{\varepsilon} < R(\phi)$ for all ϕ. Finally, notice that $R > \overline{\varepsilon}$ also implies that *every* match faces a positive probability of being destroyed for endogenous reasons. To see why, suppose $R = \varpi < \overline{\varepsilon}$; then any match that reaches a state $s > \varepsilon^{-1}(\varpi)$ will never be destroyed endogenously.

The distribution of active matches now evolves according to

$$\begin{aligned}\frac{d}{dt}\left[\left(1-u_t\right)H_t\left(x\right)\right] &= \lambda\left(1-u_t\right)\left[1-H_t\left(x\right)\right]\left[G\left(x\right)-G\left(R_t\right)\right]+\theta q\left(\theta\right)u_t\left[G\left(x\right)-G\left(R_t\right)\right] \\ &\quad -\lambda\left(1-u_t\right)H_t\left(x\right)G\left(R_t\right)-\lambda\left(1-u_t\right)H_t\left(x\right)\left[1-G\left(x\right)\right] \\ &\quad -\left(1-u_t\right)\int_R \delta\left(s\right)dH_t\left(s\right).\end{aligned} \tag{38}$$

The value functions (5) and (7) are respectively replaced by

$$\begin{aligned} rW\left(x\right) &= w\left(x\right)+\lambda\int\max\left[W\left(z\right)-U,0\right]dG\left(z\right)-\left[\delta\left(x\right)+\lambda\right]\left[W\left(x\right)-U\right] \\ rJ\left(x\right) &= \pi\left(x\right)+\lambda\int\max\left[J\left(z\right)-V,0\right]dG\left(z\right)-\left[\delta\left(x\right)+\lambda\right]\left[J\left(x\right)-V\right],\end{aligned}$$

while (4) and (6) remain unchanged. The bargaining outcome is still characterized by (8) and (9), and the value functions imply

$$S\left(x\right)=\frac{\left[\max\left(x-\phi,0\right)-c\right]k-rU+\lambda\int\max\left[S\left(z\right),0\right]dG\left(z\right)}{r+\delta\left(x\right)+\lambda},$$

which is clearly increasing in x and has a kink at ϕ, as is usual when departing from the simple fixed-cost formulation of Section 2. Plotting $S\left(x\right)$ reveals that, depending on parametrizations, two cases are possible: the kink could be above or below the horizontal axis. If it is above, then $R<\phi$ and there is equilibrium hoarding. The job-creation and destruction conditions can be derived as usual, and an equilibrium can be summarized by the $\left(\theta,R\right)$ pair that solves them. The following result provides conditions under which this model aggregates to (22), just as in the models of Section 2 and 6.1.

Proposition 3 *Suppose $\delta\left(x\right)=\delta x^{-\sigma}$ and the primitive density of shocks is*

$$dG\left(x\right)=\frac{\alpha\lambda\left(\alpha+\sigma\right)\varepsilon^{\alpha}}{\lambda\left(\alpha+\sigma\right)+\alpha\delta\varepsilon^{-\sigma}}\left(1+\frac{\delta}{\lambda}x^{-\sigma}\right)x^{-\alpha-1}$$

for $x\geq\varepsilon$ and $dG\left(x\right)=0$ otherwise, where $\varepsilon,\delta,\sigma>0$, and $\alpha>1$. Then in equilibrium, the aggregates Y, K_e and N satisfy (22).

So far I have shown that in several versions of the Mortensen-Pissarides model, for certain distributions of the idiosyncratic shocks, aggregate output looks like a Cobb-Douglas function of the aggregate labor and capital inputs. What follows generalizes the previous results by characterizing the distribution of shocks that gives rise to an aggregate CES production function.

Suppose the primitive distribution of shocks, G, is given by

$$G(x) = \begin{cases} 0 & \text{if } x < \varepsilon \\ 1 - \left[\frac{1}{\sigma}\left(\frac{x}{\varepsilon}\right)^{\frac{\rho}{1-\rho}} - \frac{1-\sigma}{\sigma}\right]^{-1/\rho} & \text{if } \varepsilon \leq x, \end{cases} \tag{39}$$

with $\varepsilon > 0$ and $\rho, \sigma \in (0,1)$.[33] Substituting (39) into (3), one sees that for any $R \geq \varepsilon$, the steady state productivity distribution of active matches is

$$H(x) = 1 - \kappa \left[\frac{1}{\sigma}\left(\frac{x}{\varepsilon}\right)^{\frac{\rho}{1-\rho}} - \frac{1-\sigma}{\sigma}\right]^{-1/\rho} \tag{40}$$

if $R \leq x$; and $H(x) = 0$ if $x < R$, with $\kappa \equiv [1 - G(R)]^{-1}$. Using a related insight due to Levhari (1968), one can establish the following result.

Proposition 4 *If the primitive distribution of the idiosyncratic shocks is given by (39), then in equilibrium, the aggregates* Y, K_e *and* N *satisfy* $Y = B\left[\sigma \bar{A} K_e^{\rho} + (1-\sigma) N^{\rho}\right]^{1/\rho}$, *with* $B = \frac{\varepsilon}{1-\sigma}$, *and* $\bar{A} = \left[\frac{1}{\sigma}\left(\frac{R}{\varepsilon}\right)^{\frac{\rho}{1-\rho}} - \frac{1-\sigma}{\sigma}\right]$.

In this case, all the characteristics of the labor market as summarized by R affect the measured productivity of inputs asymmetrically.[34] Notice that as $\rho \to 0$, (39) approaches the Pareto distribution in (20) with parameters ε and $\alpha = 1/\sigma$. So in this sense, the CES aggregate in Proposition 4 approaches the Cobb-Douglas aggregate in (22) as the elasticity of substitution $1/(1-\rho)$ approaches unity.[35]

[33] Under these conditions $G'(x) \geq 0$ and $\lim_{x\to\infty} G(x) = 1$, so G is a proper *cdf*.

[34] If the aggregation were performed using (39) instead of its truncation, then the aggregate would instead be $Y = \frac{R}{1-\sigma}\left[\sigma K_e^{\rho} + (1-\sigma) N^{\rho}\right]^{1/\rho}$. However, there is no primitive density that has (39) as its truncation.

[35] Notice, however, that the truncation of (39) does not approach (21) as $\rho \to 0$. That is, even though the primitive distribution approaches a Pareto, its truncation does not limit a truncated Pareto. This is because the density in (39) is not "closed" under truncations (as, for example, the Pareto and the exponential distributions are). This "discontinuity" introduced by the truncation is the reason why if we take the limit on the truncated *cdf* or on the CES aggregate directly, we don't obtain exactly (22).

6.3 Measurement

I conclude this section by showing how the observed level of TFP is affected by the different ways of measuring aggregate inputs that can be found in the literature. The measure of capital input used by Hall and Jones (1999) did not adjust for utilization. This means that K instead of K_e was used in the production function, which would imply $\hat{F}(K,N) = \hat{A}K^{\gamma}N^{1-\gamma}$, with $\hat{A} = \left[\frac{1-u}{1-(1-\theta)u}\right]^{\gamma} A$, as mentioned in Section 4. But in addition, Hall and Jones (1999) report they did not have data on hours per worker for all countries in their sample, so they used the number of employed workers instead of hours worked as a measure of labor input. Letting $E = 1-u$ denote employment and using (17), the number of hours worked is $N = (R/\mu)^{1/\gamma} \frac{KE}{1-(1-\theta)u}$, so their measurements of inputs imply that the aggregate relationship between inputs, output and TFP that they observed was $\tilde{F}(K,E) = \tilde{A}K^{\gamma}E^{1-\gamma}$, with $\tilde{A} = \left[\frac{(R/\mu)^{1/\gamma}K}{1-(1-\theta)u}\right]^{1-\gamma} \hat{A}$.

Finally, although the emphasis throughout the paper has been on understanding the determinants of the Solow residual, it may also be interesting to point out that the foundation for the aggregate production function provided here also has implications for other aspects of standard growth accounting exercises. In particular, according to this theory, the exponent of the capital stock in the aggregate production function, γ, is a parameter of the underlying distribution of shocks and not the share of income accruing to capital as in the standard neoclassical growth model.[36]

[36] To see that γ need not equal the capital share, let s_w and s_k denote the labor and capital shares, respectively, and consider a version of the basic model in which ϕ indexes a worker's disutility from work, $b = \tau_b k$, and there is no fixed cost (i.e., $C = 0$). Then,

$$s_w = \beta + \xi(1-\beta) - z_k \frac{K_e}{Y} + z_n \frac{N}{Y} \quad \text{and} \quad s_k = 1 - s_w,$$

where $z_k \equiv [1-(1-\xi)(1-\beta)]c - (1-\xi)[\beta c\theta + (1-\beta)\tau_b]$, $z_n \equiv (1-\xi)(1-\beta)\phi$ and ξ is an accounting parameter that denotes the fraction of the firm's rents imputed as labor income. For example, if $\xi = 1$, then $s_k = \frac{cK_e}{Y} = \frac{c}{F_1(K_e,N)}\gamma$.

7 Concluding Remarks

This paper developed a theory of TFP differences based on the interaction between institutions and the microeconomics underlying the aggregate production function. By thinking of the aggregate production function as an equilibrium relationship between aggregate inputs and output arising from the aggregation of heterogeneous micro-level production units, I was able to show analytically how the measured level of TFP – usually a "black box" that drives cross-country income differences – depends on primitive technological parameters, policies and other features of the economic environment. It seems this approach can be useful to interpret aggregate productivity data and can serve as a guide to uncover sources of cross-country differences in measured TFP.

The analysis focused on a precise class of institutions, namely, labor-market policies as measured by the magnitudes of hiring and employment subsidies, unemployment benefits and firing taxes. In the model, firm-level technologies are subject to idiosyncratic shocks that induce a cross-sectional distribution of productivities. Labor-market policies affect the productivity composition of active firms through their effects on the job-creation and destruction decisions.

Policies that make firing difficult make firms less willing to give up relatively unproductive opportunities, lowering the average productivity among active matches and aggregate TFP. Employment subsidies also make firms more tolerant of low productivity realizations, and hence they also decrease TFP. Unemployment benefits have the opposite effect. Hiring subsidies stimulate job creation and cause more competition among firms. As a result, firms become more selective and only pursue very productive ventures. The cross-sectional distribution of productivities shifts to the right, leading to a higher level of measured TFP.

A Appendix

Proof of Proposition 1.

In R-θ space, the slopes of the job-destruction and creation conditions (29) and (30) are

$$\frac{1-\beta}{\beta c}\left[1-\frac{\lambda\left(\varepsilon/R\right)^{\alpha}}{r+\delta+\lambda}\right] \quad \text{and} \quad \frac{\left(1-\beta\right)\theta q\left(\theta\right)\left(\varepsilon/R\right)^{\alpha}\tau\left(R\right)}{-c\eta\left(\theta\right)\left(r+\delta+\lambda\right)R},$$

respectively, where $\tau\left(R\right)=R+\alpha\left(r+\delta+\lambda\right)\left(\tau_h-\tau_f\right)$. Note that for all $R\geq\varepsilon$, the destruction condition slopes up and the creation condition slopes down. If $\phi=\phi_{\epsilon}$, then (29) and (30) have a unique solution, namely, $\theta\left(\phi_{\epsilon}\right)=\theta_{\epsilon}^{*}$ and $R\left(\phi_{\epsilon}\right)=\varepsilon$. Increases in ϕ only shift the destruction condition down, increasing the equilibrium level of R and decreasing the equilibrium level of θ (the creation condition is independent of ϕ). In addition, for any given ϕ, the creation condition (30) asymptotes the horizontal axis in (R,θ) space and the job-destruction condition (29) grows without bound. Therefore *(a)-(d)* follow for any $\phi>\phi_{\epsilon}$. Finally, $\phi_{\epsilon}-\varepsilon>0$ is equivalent to $\phi_{\epsilon}-R\left(\phi_{\epsilon}\right)>0$, which implies *(e)*. ■

Proof of Proposition 2.

Define $\Delta=\frac{(\varepsilon/R)^{\alpha}\tau(R)}{(r+\delta+\lambda)R}+\frac{\eta(\theta)}{\beta\theta q(\theta)}\left[1-\frac{\lambda(\varepsilon/R)^{\alpha}}{r+\delta+\lambda}\right]$. Since $\tau\left(R\right)>0$ by Proposition 1, it follows that $\Delta>0$ in any equilibrium. By totally differentiating (29) and (30),

$$\begin{aligned}\frac{\partial R}{\partial\tau_e} &= \frac{-\eta\left(\theta\right)}{\beta\theta q\left(\theta\right)\Delta}<0,\ \frac{\partial R}{\partial\tau_f}=-\left(1/\Delta\right)\left[\left(\varepsilon/R\right)^{\alpha}+\frac{r\eta\left(\theta\right)}{\beta\theta q\left(\theta\right)}\right]<0,\\ \frac{\partial R}{\partial\tau_h} &= \left(1/\Delta\right)\left(\varepsilon/R\right)^{\alpha}>0,\ \frac{\partial R}{\partial\tau_b}=-\frac{\partial R}{\partial\tau_e}>0,\end{aligned}$$

and this concludes the proof. ■

Proof of Proposition 3. First, note that the corresponding distribution function is

$$G\left(x\right)=\frac{\alpha\lambda(\alpha+\sigma)\varepsilon^{\alpha}}{\lambda(\alpha+\sigma)+\alpha\delta\varepsilon^{-\sigma}}\left\{\frac{\varepsilon^{-\alpha}}{\alpha}\left[1-\left(\frac{\varepsilon}{x}\right)^{\alpha}\right]+\frac{\delta\varepsilon^{-(\alpha+\sigma)}}{\lambda(\alpha+\sigma)}\left[1-\left(\frac{\varepsilon}{x}\right)^{\alpha+\sigma}\right]\right\}. \tag{41}$$

Verify that $G\left(\varepsilon\right)=0$ and $\lim_{x\to\infty}G\left(x\right)=1$. Under the parametric restrictions in the statement, $dG\left(x\right)\geq0$ for all x, so dG is a proper density. The restriction $\alpha>1$ ensures the mean is finite.

Imposing steady states in (38), substituting (41) and solving for $H(x)$ reveals that $H(x)$ is as in (21). ■

Proof of Proposition 4.

The problem is to find a *cdf* H that satisfies $H(R)=0$ and yields

$$Y = a\left[\sigma_1 \left(\hat{\kappa} K_e\right)^{\rho} + \sigma_2 N^{\rho}\right]^{1/\rho}, \tag{42}$$

where $\rho \in (0,1)$ and a, $\hat{\kappa}$, σ_1 and σ_2 are positive constants. Define $\varsigma(x) = \int_x zh(z)\,dz$ and $s(x) = 1 - H(x)$. Since, in general, $Y = \varsigma(\mu) K_e$ and $N = s(\mu) K_e$, (42) can be rewritten as $\varsigma(x)^{\rho} = a^{\rho}\left[\sigma_1 \hat{\kappa}^{\rho} + \sigma_2 s(x)^{\rho}\right]$. Differentiating both sides of this expression gives $\varsigma(x) = \left(\frac{x}{\sigma_2 a^{\rho}}\right)^{\frac{1}{1-\rho}} s(x)$. The last two equations yield $s(x) = \hat{\kappa}\left[\frac{1}{\sigma_1}\left(\frac{x}{\sigma_2 a}\right)^{\frac{\rho}{1-\rho}} - \frac{\sigma_2}{\sigma_1}\right]^{-1/\rho}$, which by defining

$$\varepsilon = \sigma_2 a \left(\sigma_1 + \sigma_2\right)^{\frac{1-\rho}{\rho}} \text{ and } \sigma = \frac{\sigma_1}{\sigma_1 + \sigma_2} \tag{43}$$

can be rewritten as $H(x) = 1 - \hat{\kappa}\left[\frac{1}{\sigma}\left(\frac{x}{\varepsilon}\right)^{\frac{\rho}{1-\rho}} - \frac{1-\sigma}{\sigma}\right]^{-1/\rho}$. The requirement that $H(R) = 0$ implies that $\hat{\kappa} = \kappa$ (with κ as defined in Subsection 6.2). After specifying that $H(x) = 0$ for $x < R$, this expression is identical to (40). So by construction, aggregation under (40) yields (42). And after letting $\hat{\kappa} = \kappa$ and making the substitutions in (43), one realizes that (42) is identical to the aggregate in Proposition 4. Finally, verifying that (40) is the truncation of (39) at R concludes the proof. ■

To complete the analysis of Section 5, here I report the effects of all policies on market tightness:

$$\begin{aligned}
\frac{\partial\theta}{\partial\tau_e} &= \frac{-(1-\beta)\,\theta q(\theta)\,(\varepsilon/R)^{\alpha}\,\tau(R)}{c\eta(\theta)\,(r+\delta+\lambda)\,R}\frac{\partial R}{\partial\tau_e} > 0, \; \frac{\partial\theta}{\partial\tau_h} = \frac{1-\beta}{\beta c}\left[1 - \frac{\lambda(\varepsilon/R)^{\alpha}}{r+\delta+\lambda}\right]\frac{\partial R}{\partial\tau_h} > 0, \\
\frac{\partial\theta}{\partial\tau_b} &= \frac{-(1-\beta)\,\theta q(\theta)\,(\varepsilon/R)^{\alpha}\,\tau(R)}{c\eta(\theta)\,(r+\delta+\lambda)\,R}\frac{\partial R}{\partial\tau_b} < 0, \; \frac{\partial\theta}{\partial\tau_f} = \frac{1-\beta}{\beta c}\left\{r + \left[1 - \frac{\lambda(\varepsilon/R)^{\alpha}}{r+\delta+\lambda}\right]\frac{\partial R}{\partial\tau_f}\right\}.
\end{aligned}$$

Without additional restrictions the sign of $\partial\theta/\partial\tau_f$ is ambiguous. It is negative in any equilibrium with $\phi > \phi_{\epsilon}$ if $\delta > r(1-\varepsilon)/\varepsilon$.

B Appendix

Lemma 1 *Let* $\varphi(R) = \int_R [1-G(x)]\,dx$*,* $\underline{\theta} = \frac{1-\beta}{\beta c}\left[\frac{\lambda}{r+\delta+\lambda}\varphi(0) - \phi - c\right]$*. If* τ_b *is small and* $q(\underline{\theta}) > \frac{r+\delta+\lambda}{(1-\beta)\varphi(0)}c$*, then there exists a unique pair* $(\theta, R) \in \mathbb{R}^2_+$ *that solves (15) and (16).*

Proof. Differentiating (15) and (16) respectively,

$$\begin{aligned}
\left.\frac{\partial R}{\partial \theta}\right|_{JD} &= \left\{\tfrac{(1-\beta)[1-(1-\beta)\tau_b]}{\beta c}\left[1 - \tfrac{\lambda[1-G(R)]}{r+\delta+\lambda} - \tfrac{\tau_b \beta G'(R)\varphi(R)}{[1-(1-\beta)\tau_b][1-G(R)]^2}\right]\right\}^{-1} \\
\left.\frac{\partial R}{\partial \theta}\right|_{JC} &= \frac{-(r+\delta+\lambda)\,c\eta(\theta)}{(1-\beta)\,\theta q(\theta)\,[1-G(R)]},
\end{aligned}$$

where $\eta(\theta) \equiv -\theta q'(\theta)/q(\theta) > 0$. Notice that the job-creation condition is downward sloping in (θ, R) space, while the job-destruction condition is upward sloping provided τ_b is not too big. To verify this note that if $\tau_b \approx 0$, the slope of the job-destruction condition reduces to

$$\left.\frac{\partial R}{\partial \theta}\right|_{JD} = \frac{\beta c}{(1-\beta)\left\{1 - \frac{\lambda[1-G(R)]}{r+\delta+\lambda}\right\}} > 0.$$

Therefore as long as τ_b is small enough, if an intersection exists, it must be unique. To establish existence, define

$$\begin{aligned}
T(R,\theta) &\equiv R - \tfrac{\tau_b \beta \tilde{x}(R)}{1-(1-\beta)\tau_b} - \tfrac{(1-\tau_b)(\phi+c)}{1-(1-\beta)\tau_b} - \tfrac{\beta c \theta}{(1-\beta)[1-(1-\beta)\tau_b]} + \tfrac{\lambda}{r+\delta+\lambda}\varphi(R) \\
\hat{T}(R,\theta) &\equiv \varphi(R) - \frac{(r+\delta+\lambda)\,c}{(1-\beta)\,q(\theta)}.
\end{aligned}$$

The equilibrium conditions (15) and (16) are equivalent to $T(R,\theta) = 0$ and $\hat{T}(R,\theta) = 0$, respectively. Let $\underline{\theta}$ and $\overline{\theta}$ be defined by $T(0,\underline{\theta}) = 0$ and $\hat{T}\left(0,\overline{\theta}\right) = 0$, respectively. Since the θ that satisfies (16) goes to 0 as R gets large, and the θ that satisfies (15) grows without bound as R gets large, existence is guaranteed for parametrizations that satisfy $\underline{\theta} < \overline{\theta}$. Note that $\overline{\theta}$ solves $q\left(\overline{\theta}\right) = \frac{r+\delta+\lambda}{(1-\beta)\varphi(0)}c$, and if $\tau_b \approx 0$ then $\underline{\theta}$ is as in the statement of the Lemma. Thus the condition $\underline{\theta} < \overline{\theta}$ is equivalent to the parametric restriction $q(\underline{\theta}) > \frac{r+\delta+\lambda}{(1-\beta)\varphi(0)}c$. ■

Lemma 2 *Let*

$$\underline{\theta} = \frac{(1-\beta)[1-(1-\beta)\tau_b]}{\beta c} \left\{ \left[\left(1 + \frac{\lambda}{(\alpha-1)(r+\delta+\lambda)} - \frac{\tau_b \alpha \beta}{(\alpha-1)[1-(1-\beta)\tau_b]} \right) \varepsilon - \frac{(1-\tau_b)(\phi+c)}{1-(1-\beta)\tau_b} \right] \right\}.$$

Suppose $q(\underline{\theta}) > \frac{(\alpha-1)(r+\delta+\lambda)c}{(1-\beta)\varepsilon}$, *and* $\tau_b \leq \bar{\tau}_b$, *where*

$$\bar{\tau}_b = \frac{(\alpha-1)(r+\delta)}{\beta\alpha(r+\delta+\lambda) + (1-\beta)(\alpha-1)(r+\delta)}.$$

Then there exists a unique pair $(\theta, R) \in (0, \infty) \times (\varepsilon, \infty)$ *that satisfies (24) and (25).*

Proof. Differentiating (24) and (25),

$$\begin{aligned} \left.\frac{\partial R}{\partial \theta}\right|_{JD} &= \left\{ \frac{(1-\beta)[1-(1-\beta)\tau_b]}{\beta c} \left[1 - \frac{\lambda(\varepsilon/R)^\alpha}{r+\delta+\lambda} - \frac{\tau_b\alpha\beta}{(\alpha-1)[1-(1-\beta)\tau_b]} \right] \right\}^{-1} \\ \left.\frac{\partial R}{\partial \theta}\right|_{JC} &= \frac{-(r+\delta+\lambda)c\eta(\theta)}{(1-\beta)(\varepsilon/R)^\alpha \theta q(\theta)}, \end{aligned}$$

where $\eta(\theta) \equiv -\theta q'(\theta)/q(\theta) > 0$. Thus the job-creation condition (25) is always downward sloping in (θ, R) space, while the job-destruction condition (24) is upward sloping if $\tau_b \leq \bar{\tau}_b$. So under this condition, if an intersection exists, it is unique. The parameter restriction $q(\underline{\theta}) > \frac{(\alpha-1)(r+\delta+\lambda)c}{(1-\beta)\varepsilon}$ guarantees that the two lines intersect on $(0, \infty) \times (\varepsilon, \infty)$. ■

The model of Section 2 with a general fixed cost.

The value functions are still given by (4), (5), (6) and (7), but the profit function is now

$$\pi(x) = x \min(n, k) - w(x) - ck - \phi n - C(x) k,$$

where $C(x)$ is continuous and $C' < 0$ (the inequality need only be strict for $x < \phi$). The bargaining problem is unchanged, and hours are still given by (8), so $\pi(x) = [\max(x-\phi, 0) - c - C(x)] k - w(x)$. Also from the Nash bargaining, wages are

$$w(x) = \beta [\max(x-\phi, 0) - c - C(x)] k + (1-\beta) rU, \tag{44}$$

and therefore flow profits are

$$\pi(x) = (1-\beta) \{[\max(x-\phi, 0) - c - C(x)] k - rU\}. \tag{45}$$

Steps similar to the ones that led to (14) now imply

$$S(x) = \frac{\max(x-\phi,0) - \max(R-\phi,0) + C(R) - C(x)}{r+\delta+\lambda} k. \tag{46}$$

Note that if the idiosyncratic technology shock does not induce savings in the fixed cost (i.e., if either $C(x) = 0$ for all x or $C' = 0$), then (46) reduces to $(r+\delta+\lambda) S(x) = [\max(x-\phi,0) - \max(R-\phi,0)] k$. From this it is clear that the expected surplus $S(x)$ is flat for $x < \phi$, meaning that generically, $R > \phi$, in an equilibrium with endogenous destruction. (The reservation R is indeterminate in the interval $[0,\phi]$ in the knife-edge case where the flat spot coincides with the horizontal axis.) Conversely, $C' < 0$ ensures that $S(x)$ is strictly increasing for all x, and therefore the level R for which $S(R) = 0$ may be to the right or to the left of ϕ, depending on the parametrization. In general, $S(x)$ will have a kink at $x = \phi$, but this is immaterial for my purposes. The job-destruction condition for this case is

$$p(R) - c - \frac{rU}{k} + \frac{\lambda}{r+\delta+\lambda} \hat{\varphi}(R) = 0, \tag{47}$$

where $p(R) \equiv \max(R-\phi,0) - C(R)$, rU is still given by (10), and

$$\hat{\varphi}(R) \equiv \int_R [\max(x-\phi,0) - \max(R-\phi,0) + C(R) - C(x)] \, dG(x).$$

The job-creation condition is

$$\hat{\varphi}(R) - \frac{(r+\delta+\lambda) c}{(1-\beta) q(\theta)} = 0. \tag{48}$$

Specifying the unemployment income by $b = \tau_b k$ as in Section 5, it is immediate that (47) is increasing and (48) decreasing in (θ, R) space, so they will intercept exactly once under conditions analogous to those in Lemma 1. Alternatively, letting $b = \tau_b E_G[w(x) | x \geq R]$, (47) specializes to

$$p(R) - \tfrac{(1-\tau_b)c}{1-(1-\beta)\tau_b} - \tfrac{\beta c \theta}{(1-\beta)[1-(1-\beta)\tau_b]} - \tfrac{\tau_b \beta \hat{E}(R)}{1-(1-\beta)\tau_b} + \tfrac{\lambda}{r+\delta+\lambda} \hat{\varphi}(R) = 0, \tag{49}$$

where $\hat{E}(R) \equiv [1-G(R)]^{-1} \int_R [\max(x-\phi,0) - C(x)] \, dG(x)$. For this formulation, conditions analogous to the ones in Lemma 2 (essentially a replacement rate τ_b that is not "too big") will guarantee existence and uniqueness of a pair (θ, R) that satisfies (48) and (49). Given the equilibrium (θ, R), the aggregation proceeds exactly as in Section 4.

Reinterpreting the variable cost ϕn as disutility from work.

Instead of assuming there is a variable cost of production ϕn as in Section 2, now suppose there is no such cost, but instead the worker suffers disutility ϕn if she supplies n hours to the firm. The value functions (4), (6) and (7) remain unchanged, while (5) is now

$$rW(x) = w(x) - \phi n + \lambda \int \max\left[W(z) - U, 0\right] dG(z) - (\delta + \lambda)\left[W(x) - U\right]. \tag{50}$$

The firm's flow profit is modified accordingly to $\pi(x) = x \min(n, k) - w(x) - ck - C(x)k$.[37] The bargaining solution implies a choice of hours that is still characterized by (8). Naturally, regardless of which party bears the cost ϕn, the efficient bargaining solution will ensure the efficient number of hours worked and will use the wage $w(x)$ to distribute the surplus among the partners. In fact, since $W(x)$ is now given by (50), the first-order condition (9) now yields

$$w(x) = \phi n + \beta\left[\max(x - \phi, 0) - c - C(x)\right]k + (1 - \beta) rU \tag{51}$$

$$\pi(x) = (1 - \beta)\left\{\left[\max(x - \phi, 0) - c - C(x)\right]k - rU\right\}, \tag{52}$$

where rU still satisfies (10). The outcome is that regardless of who suffers the production cost ϕn, the worker and firm share it so that the firm bears a fraction $1 - \beta$. In fact, comparing (52) with (45) confirms that the firm's flow profit is unchanged when ϕn is modeled in this way. Similarly, note that the worker's value function is also unchanged in this formulation relative to the variable cost interpretation. This can be verified by checking that substituting (51) into (50) delivers the same expression for the worker's value function as substituting (44) into (5).

[37] The fixed cost $C(x)k$ could be set to zero here. This is of no consequence for the purposes of reinterpreting ϕn as a disutility cost from work.

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