A Reply to Darby

Preston J. Miller

Monetary Adviser
Research Department
Federal Reserve Bank of Minneapolis

Thomas J. Sargent

Adviser
Research Department
Federal Reserve Bank of Minneapolis

and *Professor of Economics*
University of Minnesota

A key result of Sargent and Wallace's "Some Unpleasant Monetarist Arithmetic" (1981) is that a permanently higher government deficit must eventually be accommodated by increases in the monetary base. In "Some Pleasant Monetarist Arithmetic" (in this issue), Darby argues that this result does not currently apply to the U.S. economy because it depends on an assumption which is not supported by the data. In this reply to Darby we explain why we find his argument unconvincing and why we remain concerned about the longer-term monetary implications of high prospective federal budget deficits.

Background

Sargent and Wallace describe an economic model in which the real growth rate γ and the real interest rate r are assumed to be constants for all time. It is also assumed that monetary and budget policies initially imply a steady-state equilibrium where the real interest rate exceeds the real growth rate ($r > \gamma$). Given these assumptions, Sargent and Wallace show that any attempt to run a permanently higher deficit net-of-interest is simply not feasible unless the supply of base money is eventually increased. Without an eventual increase in the base-money supply, a permanent increase in the deficit would cause the ratio of interest-bearing government bonds to national income to diverge to infinity (see Figure 1), so at some point that ratio would outstrip the ratio of total wealth to income. That is, the government would eventually be unable to command the resources needed to pay its debt.

Darby's model retains Sargent and Wallace's assumptions that the real interest rate and real growth rate are constants, but it departs from their model by assuming that the real growth rate is greater than the real interest rate ($\gamma > r$). As a result, the consequences of a permanent increase in the deficit are very different from those claimed by Sargent and Wallace. Under Darby's assumption that $\gamma > r$, a permanent increase in the deficit is feasible even with no change in the supply of base money from its initial path. Such a policy change simply leads to a higher eventual level for the ratio of interest-bearing government bonds to national income (see Figure 2). Although the level of the bond-to-income ratio may be higher, the government can always command the resources needed to pay its debt.

Darby's Evidence

Darby examines U.S. data to determine whether Sargent and Wallace's or his own model is more appropriate as a description of the current situation facing the United States. His evidence, simple and direct, rests on taking the formal Sargent-Wallace model and his own version of their model quite literally. Since both models assume a constant real interest rate and real growth rate, the sign of the difference between these constant rates is the critical distinction between the two models. Darby's procedure is simply to compute the historical average of the difference between the real growth rate and the real interest rate. He interprets the sign of this difference as settling the matter: if $\gamma > r$ so that $\gamma - r$ is positive, Darby's model applies; if $r > \gamma$ so that $\gamma - r$ is negative, Sargent-Wallace's applies. As long as the formal models explicitly analyzed by Sargent-Wallace and Darby are taken literally, Darby's test is the natural one to perform.

Figures 1 and 2

Financing a Higher Deficit Without Increasing the Supply of Base Money

Effect on Federal Bonds Held by the Public (as a percentage of GNP)

Figure 1 **With Sargent-Wallace Assumption** ($r > \gamma$)

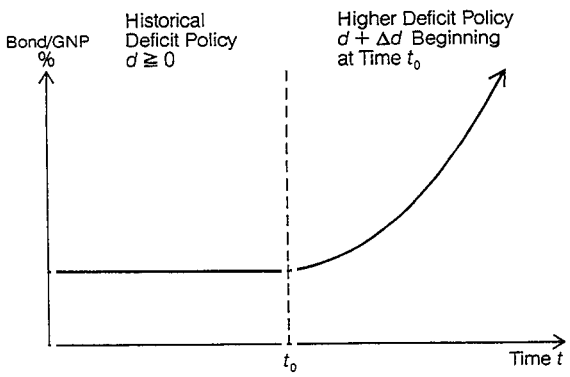
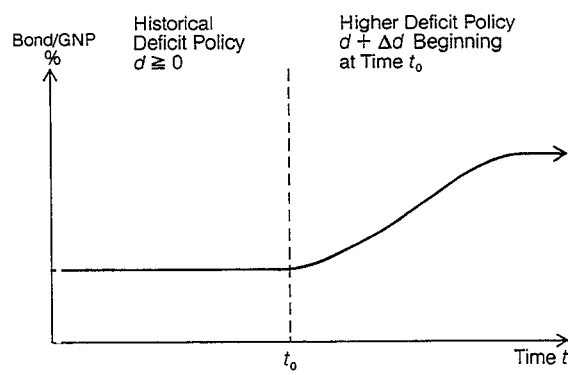


Figure 2 **With Darby Assumption** ($\gamma > r$)



Is Darby's Evidence Sufficient?

Our own interpretation of Darby's evidence, however, is conditioned by our preference not to take literally Sargent and Wallace's assumption that the real interest rate is constant and, therefore, independent of budget policy. Instead, Sargent and Wallace can be regarded as using their model to approximate a more complicated one in which the real interest rate is itself a function of the budget and monetary policies in place.[1] Their intent was to point out the choices facing monetary policymakers whenever budget policies cause the real interest rate to exceed the real growth rate. Such was the situation when Sargent and Wallace wrote their paper in the first half of 1981, a time when the real interest rate exceeded the real growth rate and when monetary and budget policymakers were announcing moves that seemed to imply that the real interest rate would continue to exceed the real growth rate for a sustained period.

In models more general than the one analyzed by Sargent and Wallace, the real interest rate on government bonds is not a constant but is partly determined as a rising function of the ratio of interest-bearing government bonds to base money. In such models, the difference between the real growth rate and real interest rate depends on the monetary and budget policies in place. From the perspective of these more general models, Darby's empirical evidence does not resolve the matter of the relevance of Sargent and Wallace's argument for the choices currently facing U.S. policymakers.

Viewed through such models, evidence that the real interest rate has averaged less than the real growth rate over some historical period may be interpreted as partly reflecting the monetary and budget policies in place during that historical period. The average difference between the real growth rate and the real interest rate would not be expected to remain the same after a change in monetary and budget policies, such as a switch to a regime with higher average deficits.[2] Moreover, in the context of more general models in which the real interest rate depends on monetary and budget policies, much more is involved than simply comparing a real growth rate with a real interest rate to determine the feasibility of financing a permanent increase in the deficit without resorting to printing more base money.

[1] In their concluding remarks, Sargent and Wallace indicate that they had in mind a model where the real interest rate depends on policy. They state that their assumption of a constant real interest rate was made "because it seems to be maintained by many of those who argue for a low rate of growth of money no matter how big the current deficit is. If we were to replace that assumption, we would instead assume that the public's demand for government bonds is an increasing function of the real rate of return on bonds . . ." (p. 6).

[2] That the distribution of the sequence of equilibrium real interest rates depends on budget and monetary policies is an application of the principles underlying the Lucas critique to the problem at hand. See Lucas 1976.

More Is Involved

In order to show that more is involved, we make use of a simple form of the steady-state budget constraint:[3]

$$(1) \qquad d = (\pi+\gamma)m(r, \pi) + (\gamma-r)b(r, \pi)$$

where the following are ratios to gross national product (GNP),

d = deficit net-of-interest

m = demand for base money

b = demand for government bonds

and where the following are rates per unit of time,

π = inflation

γ = real growth

r = real interest.

We assume the markets for money and bonds clear (that is, the supply of money and bonds equals the demand), so that

$$(2) \qquad m(r, \pi) = M_t/Y_t \text{ and } b(r, \pi) = B_t/Y_t$$

where

M = stock of base money

B = stock of government bonds

Y = nominal GNP

and t subscripts denote a given time period. The steady-state budget constraint states that the deficit net-of-interest d must be financed by seignorage on money $(\pi+\gamma)m(r, \pi)$ and by seignorage on bonds $(\gamma-r)b(r, \pi)$. (*Seignorage* is the revenue the government earns from issuing money and bonds.) A deficit policy is feasible only when the government can earn enough seignorage on its money and bond issue to finance its deficit net-of-interest.

The steady-state budget constraint helps to make precise our analysis of the insufficiency of Darby's empirical evidence and to suggest what empirical evidence is sufficient in the context of more general models. Suppose the economy is initially in equilibrium such that, given a deficit net-of-interest d and a path of money M over time, the money-to-GNP and bond-to-GNP ratios (M_t/Y_t and B_t/Y_t) and the inflation rate, real growth rate, and real interest rate (π, γ, and r) are all constant over time. The issue is whether it is possible to raise the deficit net-of-interest for all time to a new level $d + \Delta d$ while keeping the path of money fixed at $\{M_t\}$ and have the economy converge to a new equilibrium where the ratios M_t'/Y_t' and B_t'/Y_t' and the rates π', γ', and r' once again are all constant over time. From the steady-state budget constraint this is possible only if

$$(3) \qquad d + \Delta d = (\pi'+\gamma')m(r', \pi') + (\gamma'-r')b(r', \pi').$$

Equation (3) says that the deficit net-of-interest can be permanently raised to a higher level only if the total seignorage from money and bonds calculated at the new equilibrium rates (r', γ', and π', consistent with a fixed path of money) is enough to cover the higher deficit.

In general, to determine empirically if it is possible to finance a higher deficit net-of-interest $d+\Delta d$ by bond issue alone, we would require estimates of the elasticities of money demand and bond demand (in order to determine how these demands change when the rates of real interest and inflation change to r' and π'). We would also need an estimate of how the economy's real growth rate varies with the rates of real interest and inflation. It would then be possible to determine quantitatively how the seignorage represented on the right side of the steady-state budget constraint changes when the deficit net-of-interest on the left side changes.

Since the equilibrium rates depend on the policies in place, it is not sufficient to examine the average difference between the rates of real growth and real interest $(\bar{\gamma}-\bar{r})$ over some extended historical period, as Darby does. If historically the government has been running a deficit d, then $\bar{\gamma}$ and $\bar{r}$ will reflect the rates corresponding to that policy—the rates γ and r in equation (1). In order to determine the feasibility of financing a higher deficit $d + \Delta d$ by bond issue alone, however, estimates of γ' and r' are required—as shown in equation (3). Given a fixed path for the monetary base, a proposed policy of running a persistently higher deficit net-of-interest than has been observed historically would be expected to raise the real interest rate relative to the real growth rate, thereby shifting their relationship toward that supposed by Sargent and Wallace.

Another reason that simply looking at the difference between $\bar{\gamma}$ and $\bar{r}$ does not suffice is because the change

[3]The constraint is derived in Miller 1983, pp. 12–13. Its notation has been altered slightly here to conform with Darby's.

Figures 3 and 4

Two Cases Where Darby's Evidence is Insufficient When r Rises With the Ratio of Bonds to Money

Figure 3 **Case One:** $\bar{r} \geq r^*$

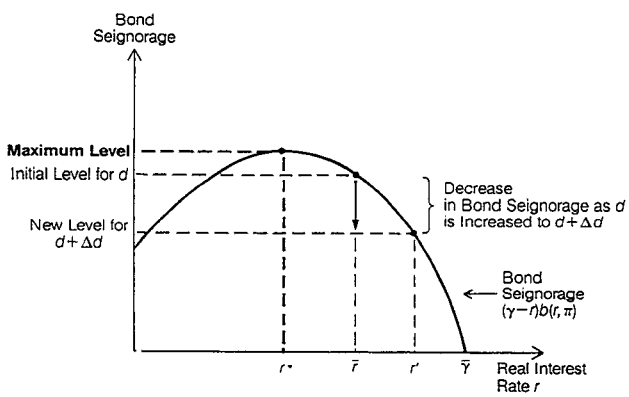
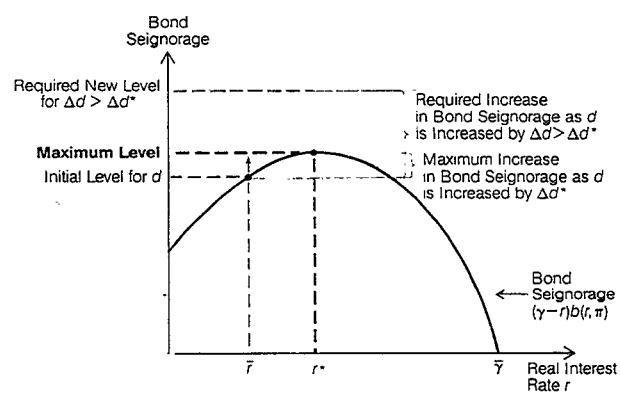


Figure 4 **Case Two:** $\bar{r} < r^*$



in seignorage caused by a change in deficit policy depends on other factors—such as how the inflation rate changes and how the new rates of interest and inflation affect the demands for money and bonds (see equation [3]). For example, an increase in the real interest rate could generally be expected to cause people to shift their asset holdings from money to bonds, thereby lowering the demand for money while raising the demand for bonds.

A Model That Shows Darby's Evidence Isn't Sufficient

In order to illustrate that Darby's evidence is not sufficent, we consider a simple monetarist model, which is taken from our more general class where the real interest rate rises with the ratio of bonds to money. This model (see Miller 1982) has the following features:

a. The real growth rate γ is fixed.

b. The inflation rate π is determined by the growth in the stock of base money.[4]

c. The real interest rate r is inversely related to the stock of private capital.

From the steady-state budget constraint, the change in seignorage as the deficit d is increased to the new level $d+\Delta d$ while the path of money is held constant is given by

$$(4) \qquad \Delta \text{ seignorage} \cong t_B \Delta b + \Delta t_B b$$

where

$t_B = \gamma - r$, the rate of seignorage on bonds

$\Delta t_B = r - r'$, the change in the rate of seignorage on bonds

$\Delta b = b(r', \pi) - b(r, \pi)$, the change in the demand for bonds.

Such a change in policy is feasible only if the seignorage increases by the same amount as the deficit.

In this model such a change in deficit policy has both a quantity and a price effect on bond seignorage ($t_B \Delta b$ and $\Delta t_B b$, respectively). When $\gamma > r$, the quantity effect is positive as the real demand for bonds increases at a positive seignorage rate ($\gamma - r$). The price effect, however,

[4]We assume a simple quantity demand function for money so that $m_1 = 0$.

is negative as additional government debt drives out private capital and raises the real interest rate at which the government must borrow. In this model there is a maximum amount of bond seignorage that can be generated. For the additional deficit to be financed by bond issue alone, it is necessary that the seignorage from bonds initially be less than that maximum. Because the quantity and price effects are of opposite sign, seignorage is maximized at some real interest rate $r^* < \gamma$, as shown in Figures 3 and 4.

Two Cases

This simple model indicates two cases where we could observe historically that $\bar{\gamma} > \bar{r}$ and where it still would not be possible to finance a permanent increase in the deficit net-of-interest by bond issue alone. The first case is when the real interest rate is greater than or equal to the real interest rate at which seignorage from bonds is maximized: $\bar{r} \geq r^*$. In this case, any further increase in bond issue will cause the seignorage from bonds to decrease (see Figure 3). So even though $\bar{\gamma} > \bar{r}$, it would not be possible to finance any increase in deficits by bond issue alone. The second case is when the real interest rate is less than the rate at which seignorage from bonds is maximized, $\bar{r} < r^*$, but the increase in the deficit is too large. Since there is a finite maximum to bond seignorage, it is always possible that the increase in the deficit will exceed the maximum increase in bond seignorage (see Figure 4).

Which Arithmetic Best Applies Currently?

Current observations of the relationship between the real interest rate and real growth rate seem consistent with the implications of our simple model and the more general models of which ours is an example. It appears that since 1980, the real interest rate has tended to rise relative to the real growth rate as it became increasingly evident that the government had embarked on a historically unprecedented policy of large and persistent deficits net-of-interest. With nominal interest rates currently far above the inflation rates predicted for and realized in 1984, the real interest rate seems to be high and the difference between the real growth rate and real interest rate ($\gamma - r$) seems to be *negative.* These observations favor Sargent and Wallace's position.

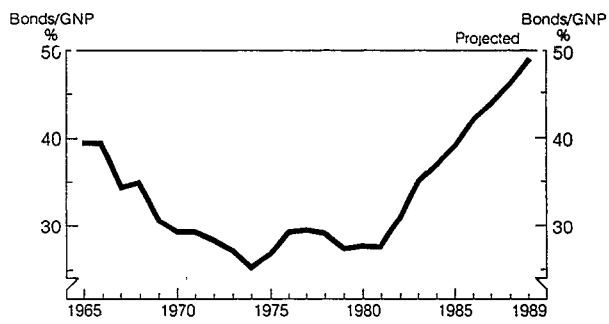
Perhaps more direct evidence favoring Sargent and Wallace's position is contained in recent Congressional Budget Office (CBO) projections of a steady rise in the government's bond-to-GNP ratio (see Figure 5 and compare with Figures 1 and 2). The CBO projects this ratio assuming that the budget policies in place at the beginning of 1984 are maintained and that monetary policy does not accommodate the deficits (U.S. Congress 1984). Even under its assumption of modestly declining real interest rates (which already seems overly optimistic), the CBO's projected bond-to-GNP ratio rises steadily.

Because of the apparent error in the CBO's assumption for the real interest rate, the bond-to-GNP ratio would still be expected to rise for real growth rates higher than the roughly 3.5 percent assumed by the CBO (U.S. Congress 1984, p. 12). Thus, even under more optimistic assumptions about real growth, current deficit policy would still not be sustainable without future monetary accommodation.

Figure 5

CBO Projections of the Bond-to-Income Ratio

Federal Bonds Held by the Public
(as a percentage of GNP)*



*Plotted for the end of fiscal years
Source: CBO (U.S. Congress 1984, p.10)

References

Lucas, Robert E. 1976. Econometric policy evaluation: A critique. In *The Phillips curve and labor markets,* ed. K. Brunner and A. H. Melzer. Carnegie-Rochester Conference Series on Public Policy 1: 19–46. Amsterdam: North-Holland.

Miller, Preston J. 1982. A monetarist approach to federal budget control. Research Department Working Paper 210. Federal Reserve Bank of Minneapolis.

_____. 1983. Budget deficit mythology. *Federal Reserve Bank of Minneapolis Quarterly Review* 7 (Fall): 1–13.

Sargent, Thomas J., and Wallace, Neil. 1981. Some unpleasant monetarist arithmetic. *Federal Reserve Bank of Minneapolis Quarterly Review* 5 (Fall): 1–17.

U.S. Congress. 1984. Congressional Budget Office. Baseline budget projections for fiscal years 1985–1989. A report to the Senate and House committees on the budget—Part II. Washington, D.C.: U.S. Government Printing Office.

The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.