Using Vector Autoregressions to Measure the Uncertainty in Minnesota's Revenue Forecasts

Robert B. Litterman
Economist

Thomas M. Supel
Senior Economist

Research Department
Federal Reserve Bank of Minneapolis

As fiscal years 1981 and 1982 progressed, the state of Minnesota faced the unpleasant prospect of a growing budget deficit. Revenues were less than those expected at the time the legislature enacted its spending program. By the end of fiscal year 1982, actual revenues were approximately 9 percent below an estimate made as late as August 5, 1981. Spending programs typically had been geared to estimated revenues, so a broad spectrum of the state's citizenry had to bear the burden of program retrenchment due to the revenue shortfall.

In an attempt to understand the shortfall problem, Minnesota's governor appointed a task force in the fall of 1981 to review the state's fiscal planning procedures. The task force recognized that the fiscal planning process involves two vital ingredients. One ingredient is an accurate forecasting procedure, one that describes the actual path of revenues over time with an average error as small as possible. Because even the best currently known forecasting procedures can still produce errors of a serious magnitude from time to time, however, adequate fiscal planning also requires an estimate of the likely size of errors in the forecast. The task force found that Minnesota's fiscal difficulties had arisen in large part from inadequate contingency planning for such errors and recommended that

> in addition to providing a "most likely" forecast [of revenues] the Commissioner of Finance should provide a measure of possible variation to facilitate preparation of contingency plans (Minnesota, Office of the Governor 1981, p. 12).

The task force, unfortunately, did not tell the state how to produce a measure of possible variation. Our research indicates that adding a procedure known as vector autoregression (VAR)[1] to its current forecasting procedure would permit the state of Minnesota to at least maintain the accuracy of its forecasts while generating a more objective measure of the uncertainty in the forecasting procedure than the current procedure provides.[2]

Adding VAR Improves Accuracy

The success of VAR or similar procedures in a wide range of contexts (see the box on pp. 14–15) formed much of our motivation to explore the use of this procedure in forecasting Minnesota's tax receipts. Our research indicates that when VAR is used in conjunction with Minnesota's current revenue forecasting procedure, the combined

[1] We will not describe the details of the VAR forecasting procedure in this paper. The interested reader may wish to refer to Litterman 1979 and 1982 or Sargent 1979. For purposes of this discussion, it suffices to think of a set of variables, such as sales tax revenues and personal income, that are relevant to the state in generating forecasts of tax receipts. Then take each of these variables and relate its performance to the past values of itself as well as to the past values of all (a vector) the other variables.

[2] Our concern with revenue forecasting is primarily methodological, and we believe that our comments are relevant to many users of economic forecasts. Minnesota is used throughout this discussion as an example only because we happen to be familiar with its revenue problems and the methods used to generate its forecasts.

procedures can provide better estimates of the "most likely" revenues and thereby provide a stronger foundation for Minnesota's fiscal planning than the current procedure can. The accuracy of our VAR procedure is quantified in Table 1, which indicates that although the errors may at times seem disturbingly large, they are, in general, no worse than those generated by Minnesota's current procedure.

In order to compare forecasts produced by Minnesota's current procedure with those generated by our VAR procedure, we had to make some adjustments and compromises. One compromise was in the forecast horizon at which we compared the two procedures. For Minnesota's fiscal contingency planning, a comparison of the two procedures' revenue forecasts for upcoming biennia would probably have been most relevant, but we could not find a record of the state's biennial revenue forecasts. So instead of comparing biennial forecasts from the two procedures, we compared their forecasts of revenues for the current and following fiscal years, for which we were able to construct an adequate (though not complete) history of Minnesota's forecasts.

To make a fair comparison of these fiscal year forecasts, we had to base our VAR forecasts on roughly the same information that was available to the state when it made its revenue forecasts. Otherwise, because additional information tends to improve the forecasts of any procedure, one procedure might have seemed better than the

Table 1

Both VAR and the state's procedure can produce serious errors . . .

Errors in Forecasts of Minnesota Tax Revenues for Selected Periods*

	Forecasts for the Current Fiscal Year				Forecasts for the Following Fiscal Year			
	Individual Income Tax Receipts		Total Tax Receipts		Individual Income Tax Receipts		Total Tax Receipts	
Fiscal Years †	VAR	State	VAR	State	VAR	State	VAR	State
1971	-3.4	n.a.	-1.1	n.a.	-13.9	6.1	-6.2	11.2
1972	0.4	n.a.	0.2	n.a.	-4.8	n.a.	1.6	n.a.
1973	0.2	3.0	-0.4	2.1	-2.9	-9.0	-6.3	-10.3
1974	-1.2	-0.6	-1.0	-1.5	-1.3	-8.7	-1.3	-7.2
1975	0.6	-1.6	0.3	-0.7	2.2	3.7	-2.2	0.1
1976	-0.8	n.a.	-0.5	n.a.	0.3	n.a.	-3.2	n.a.
1977	2.7	-1.4	1.5	-2.4	5.7	-0.2	1.3	-1.7
1978	-0.4	n.a.	-0.4	n.a.	-1.8	-3.5	-3.9	-5.4
1979	-2.7	-0.6	-2.8	-1.6	2.9	1.2	0.5	0.0
1980	3.0	2.1	2.4	3.6	6.9	-0.8	8.8	2.8
1981	0.5	n.a.	0.6	n.a.	13.5	5.8	11.6	8.9
1982	6.0	n.a.	3.4	n.a.	n.a.	n.a.	n.a.	n.a.
Mean Absolute Error	1.8	1.6	1.2	2.0	5.1	4.3	4.3	5.3

n.a. = not available

*The difference between the forecasted and actual levels as a percentage of the actual level.

† All forecasts were made late in the fiscal year, which runs from July 1 to June 30.

Sources of basic data: Minnesota Department of Revenue, Research Office and Tax Research Division, various dates; Minnesota Department of Taxation, various dates

other simply because it incorporated more information.

New economic information becomes available every day, so one way that we tried to keep information constant between the two procedures was to compare forecasts made on nearly the same dates. (See the Appendix for an illustration of the effect of additional data on VAR's forecasting accuracy.) Because our VAR forecasts were based on data for calendar quarters while Minnesota's forecasts were not always computed when this data first became available, we could not maintain exact equivalence in the information available to the two procedures. The best we could do was to match nearby forecast dates for the two procedures while giving the state's procedure the benefit of whatever advantage we could not eliminate.

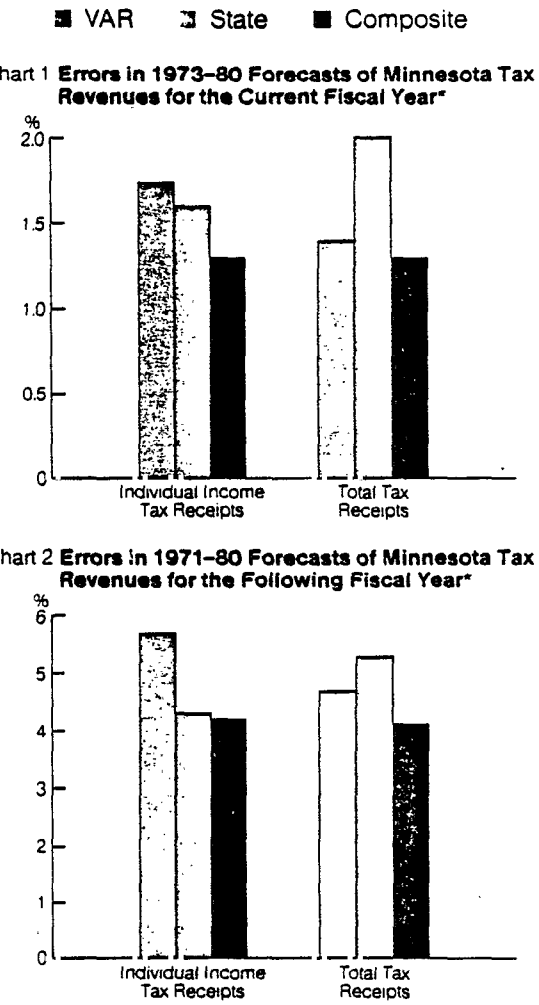
We also discovered that information about tax law changes was very important. Since 1967 prior to each major change in Minnesota's tax laws, the Finance (earlier the Revenue) Department has adjusted its revenue forecasts by an estimate of the effects of the tax law change. When we added a variable representing these adjustments to our VAR, its average forecast error was substantially reduced (see the Appendix). Since Minnesota's forecasts also incorporated these adjustments, we chose to compare those forecasts with forecasts from a version of our VAR that included the tax law adjustment variables listed in the Appendix.

The results in Table 1 and Charts 1 and 2 show that neither procedure was clearly more accurate than the other and that each was less accurate than a combination of the two. In terms of the criterion of mean absolute error, the state's procedure was slightly better than our VAR procedure in forecasting individual income taxes for the current and following fiscal years. But for total tax receipts, our VAR did a somewhat better job than the state procedure for both forecast horizons. Charts 1 and 2 show that in each case a composite forecast—one that averaged the state and the VAR forecasts in each period—produced a smaller average error than either the VAR or the state procedure alone.

Even when VAR is added to the state procedure, however, errors can still be of a serious magnitude. For this reason, if accuracy were the only criterion for adopting a forecasting procedure, we do not feel that the case for VAR would be especially strong. However, we think two other important considerations weigh in favor of the VAR forecasting procedure. For one thing, it is much cheaper than the current state procedure, costing only about one dollar of computer time to generate a single biennium

Charts 1 and 2

. . . but composite forecasts, produced by averaging VAR's forecasts with those of the state, are more accurate than forecasts produced by either procedure alone.



*Mean absolute value of the following: the difference between the forecasted and actual levels as a percentage of the actual level computed for the periods for which information was available for both VAR and the state. All forecasts were made late in the fiscal year.

Sources of basic data: Minnesota Department of Revenue, Research Office and Tax Research Division, various dates; Minnesota Department of Taxation, various dates

forecast versus thousands of dollars to do a complete state simulation. More importantly, the real virtue of the VAR procedure is its ability to generate an objective estimate of the uncertainty in the forecasting procedure.

VAR Also Quantifies Uncertainty

In planning for possible revenue shortfalls, it is important to objectively measure the uncertainty in revenue forecasting procedures. An objective measurement is possible with a VAR procedure but not with Minnesota's current procedure.

Measuring Uncertainty Is Important

Decisionmakers need objective estimates of the probability that revenues will fall short of projections by any given amount. Without this quantification of the risks of shortfalls, decisionmakers can't rationally weigh the probable costs of shortfalls against the certain costs of actions that could be taken to prevent shortfalls.

For example, Minnesota's recent experience suggests that revenue shortfalls are indeed costly. Although it is difficult to attach dollar values to these costs, residents suffered disruptions and cutbacks in a wide range of critical programs, such as education, highways, and social services. In addition, reduced investor confidence about the state's fiscal management resulted in higher interest rates on state obligations and possibly inhibited investors' willingness to undertake corporate activity in the state.

Although revenue shortfalls are costly, this does not necessarily imply that they should be completely avoided. Actions that would reduce the chance of a shortfall, such as larger student/teacher ratios in the public schools or higher taxes, obviously have immediate and certain costs of their own. Therefore in their fiscal contingency planning, decisionmakers must weigh the costs of revenue shortfalls against the costs of such actions.

In order to balance the potential costs of shortfalls against the immediate costs of preventive actions, however, decisionmakers need estimates of the chances of revenue shortfalls. One of the programs Minnesota has adopted to prevent or alleviate revenue shortfalls—allocating revenue to a reserve fund (that is, a state savings account)—illustrates this point.[3] Under such a program, citizens bear an immediate tax burden to protect themselves against the possibility of a shortfall that may or may not occur in the near future. It is important, therefore, that the fund level is not excessively costly relative to the benefits to be derived from the fund.

One important ingredient in the decision about the size of the reserve fund is a quantitative estimate of the chances of various shortfalls occurring. For example, it has been suggested that Minnesota's reserve fund be set at $500 million. An immediate question is why $500 million? Why not $1 billion or $4 billion? The $4 billion option is fairly easy to rule out. This is roughly half of Minnesota's biennial budget, and even with all the difficulties inherent in forecasting, shortfalls this large have a virtually zero chance of occurring. Setting the reserve fund at $4 billion would be much like paying an annual premium of $25,000 for flood insurance on a $50,000 house in an area where severe floods occur about once a century. But the choice between $500 million and $1 billion is more difficult. According to our estimates, shortfalls of both these magnitudes have significant chances of occurring, although a shortfall of $500 million is, of course, much more likely than one of $1 billion. Thus, the choice between these two magnitudes depends upon the quantitative meaning of "chance of occurring."

Once uncertainty is quantified so that the chance of a shortfall of any given size is known, the decisionmakers can choose a reserve fund level that is consistent with the risk they are willing to assume that the fund will not in fact cover the shortfall. Without this quantitative measure of the uncertainty in the revenue forecasting procedure, the level of the reserve fund will be set without even an approximately accurate measure of its adequacy. Such a fund might be much too small so that there are frequent program disruptions and a poor bond rating, or the fund might be much too large with citizens paying tax bills that exceed the benefits they derive from the fund.

The Current Procedure Can't Measure Uncertainty

Despite the importance of a quantitative measure of the uncertainty in the revenue forecasting procedure, Minnesota's current forecasting procedure cannot generate this information. Minnesota's revenue forecasting begins with a forecast of the national economy that is supplied by a consulting service. Then the staff of the Minnesota Finance Department translates this national forecast into a forecast of state revenues. In order to quantify the

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[3] The research this paper is based on was completed before Minnesota adopted a package of contingency plans, including a $250 million reserve fund, for dealing with revenue shortfalls. We analyzed instead an earlier proposal for simply a $500 million reserve fund. Since our purpose is primarily to illustrate the capabilities of the VAR procedure rather than to analyze specific Minnesota contingency plans, we have not recomputed our examples to incorporate the $250 million reserve fund.

uncertainty in the resulting revenue forecast, Minnesota would, at minimum, need meaningful quantitative measures of the uncertainty in both the national forecasting procedure and the translation process. The state has neither of these important pieces of information.

The consulting service's national forecast is generated by a judgmentally managed large-scale macroeconomic model, which, by its very nature, cannot generate objective probability statements about the performance of the economy.[4] We describe this model as judgmentally managed because the forecasts that it generates are, in general, not acceptable to its human managers. Therefore the managers alter the forecasts of the model to more closely match their judgment about the future course of the economy.

Managers impose their judgment on the output of large-scale macro models because the managers typically have little choice. Without management, these models frequently produce nonsensical descriptions of the economy. A recent study for the Joint Economic Committee recognized this problem when it stated that

> the pure simulations show that as they now are, the models [of three leading consulting firms] cannot be used by themselves—that is, without management—to decide what money growth strategy is optimal in the long run; not even whether 10 percent growth per year will bring happier results than zero growth. The pure simulations produce a variety of puzzling results which suggest that none of the models, as now delineated, defines the links between money growth and other key macroeconomic variables well enough to resolve these questions without management (Weintraub 1982, p. 2).

Once a manager's judgment about the outcome is imposed on the model's forecast, the model itself cannot provide an objective probability distribution of outcomes. It becomes instead essentially an elaborate accounting device. If a path for the gross national product (GNP) that the manager deems reasonable is imposed, the model is capable of quickly generating projections for the components of GNP that are consistent in the sense of adding up to the right total. But once the manager's judgment is imposed, neither the model nor its manager is able to generate objective probabilities of events that concern decisionmakers. Consequently, most forecasting services provide a variety of forecast paths or scenarios and attach probabilities to these scenarios with the clear caveat that the probabilities are subjective and are based on the manager's personal judgment.

Some Examples of the Accuracy of VAR and Similar Procedures

A variety of previous research efforts have shown that VAR, or closely related, procedures can generate relatively accurate forecasts. Forecasts of the national economy, Canadian prices, and Iowa state revenues provide examples of the variety of economic contexts in which VAR-type procedures have been successfully implemented.

In the case of the national economy, many private firms produce forecasts by combining subjective judgments with predictions from large-scale macroeconomic models, and VAR forecasts compare favorably with a sampling of these entrepreneurial efforts. In the accompanying charts, we compare the forecast performance of our VAR procedure over forecast horizons ranging from one to eight quarters with the performances of several of these judgmentally managed large-scale macro models. Although the number of forecasts used to compare the models is too small to support firm conclusions, our VAR procedure seems to forecast real gross national product (GNP) and the unemployment rate better than the other procedures (according to the criterion of mean absolute error) when the forecast horizon is four or more quarters ahead. Its relative performance is weaker when forecasting these variables for only one quarter ahead. In terms of the rate of inflation (as measured by the implicit GNP deflator) and the Treasury bill rate, the results are mixed with VAR doing better than some macro models but worse than others.

Similar results appeared when predictions of the Canadian consumer price index produced by a VAR-type procedure were compared with the predictions of 12 financial institutions that regularly forecast this measure. Three common accuracy criteria were used: percentage mean absolute error, rank sum, and percentage root mean square error. The relative accuracy of the VAR-type procedure was evident as it ranked first, tied for second, and placed fourth, respectively (Gupta 1982).

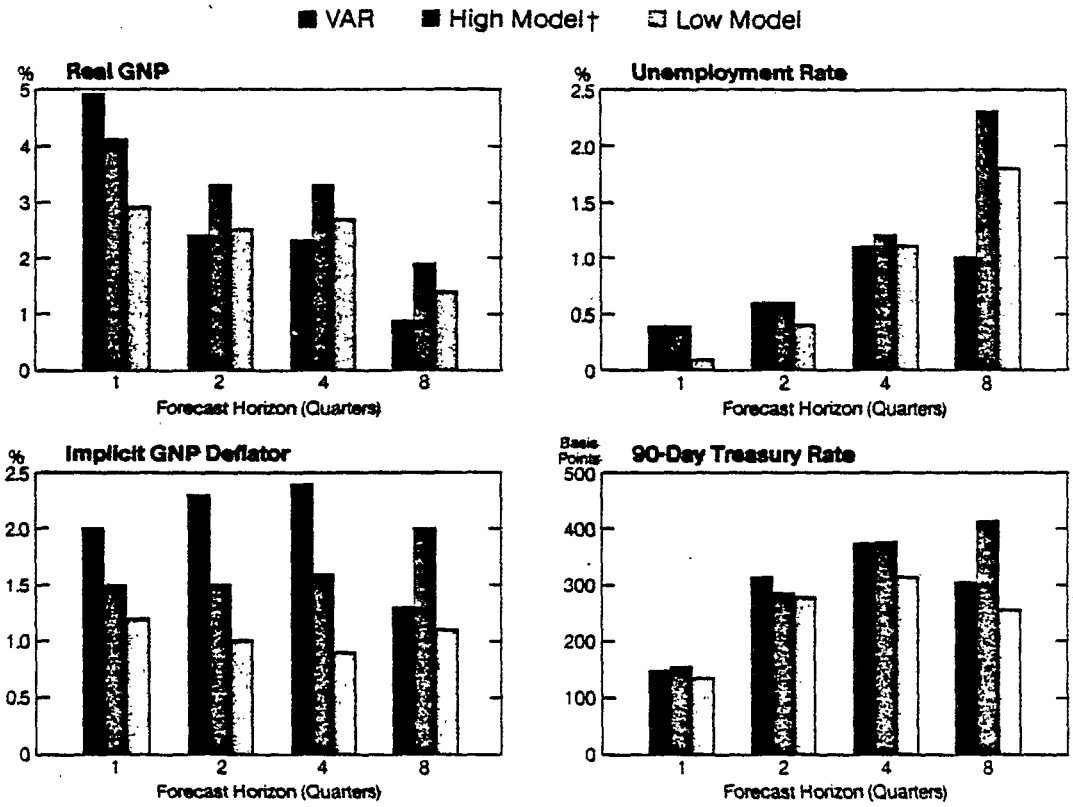
Barnard and Dent's model of Iowa tax receipts also demonstrated the relative accuracy of VAR-type procedures. The revenue forecast generated by their model compared favorably with the forecast produced by the Iowa tax authorities using ad hoc procedures (Barnard and Dent 1979).

It turns out that translating these scenarios and subjective probabilities into useful statements about the probabil-

[4]This problem seems to be common to forecasts provided by all the leading consulting services, so our comments should not be interpreted as a recommendation for or against any particular consulting service.

VAR's forecasts for the national economy compare favorably with those of large-scale macroeconomic models.

Errors in Forecasts of National Variables
From the Second Quarter of 1980 to the Fourth Quarter of 1982*



*Mean absolute value of the following: the difference between forecasted and actual growth (both at annualized percentage rates) from the time the forecast was made to the forecasted period. It should be noted that these results are based on a very small sample of large-scale macroeconomic models—three models in the case of the Treasury bill rate and five models for the other variables.

All forecasts were made early in the quarter, that is, after the first release of the National Income Accounts data for the previous quarter.

† *High model* refers to the large-scale macroeconomic model forecast with the largest error for each time period, and *low model* refers to the large-scale macroeconomic model forecast with the smallest error for each time period. Thus, these labels may not always pertain to the same model.

Source: Unpublished data of Stephen McNees, used with his kind permission

ities of state tax receipts is not possible because the scenarios do not provide the necessary detail and precision. Although an intuitively appealing interpretation of the scenarios and subjective probabilities is that they provide information about the range of possible state revenue outcomes, we are unable to construct any reasonable interpretation in which they do in fact provide useful information.

Our attempt to interpret the subjective probabilities begins with some assumptions. We assume that no one

takes the subjective probability statements literally. That is, when the consulting service presents a few scenarios whose probabilities sum to one, it does not mean to imply, in our judgment, that these are the only possible economic outcomes that can occur over the forecast horizon. Consequently, we further assume that what the consulting service really means to convey by its subjective probability statements is that there is some range around each scenario and that the subjective probability statements quantify the possibility of an outcome in this range.

Unfortunately, the consulting service does not provide enough detail or precision in its scenarios and subjective probabilities. Given the way Minnesota uses the scenarios, however, such detailed and precise information is critical to translating the scenarios and subjective probabilities into useful measures of the uncertainty in the revenue forecasting procedure. The following simplified version of Minnesota's forecasting procedure illustrates this point.

Although there are some elements of indexing, the Minnesota tax system is for the most part *value-based*, wherein revenues are determined primarily by tax rates applied to the nominal or current dollar value of income or spending. For purposes of this discussion, let's simplify matters by supposing that Minnesota tax revenue growth is determined only by the growth in the nominal value of GNP and the growth in the effective tax rate, or

$$tr = t + ngnp$$

where

tr = percentage change in state tax revenues

t = percentage change in the effective tax rate

and

$ngnp$ = percentage change in nominal GNP.

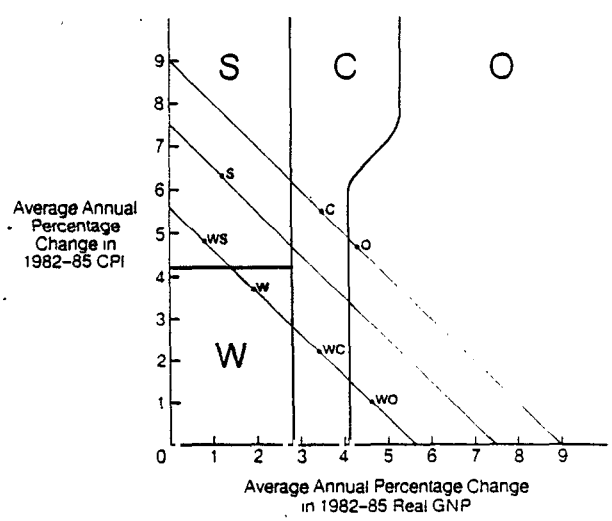
This simple representation of the Minnesota tax system can be used to illustrate the procedure used by the Minnesota Finance Department to forecast state revenues. The consulting service provides an estimate of *ngnp*, and the Finance Department then translates that national forecast into an estimate of state revenues. The "translator," t in this case, is projected by a complex set of procedures and judgments made by the Finance Department staff with the assistance of several other agencies.

Even though the forecasted value of t in our simplified setup is the result of many complex decisions, let's assume for the time being that the Finance Department staff knows the value of t with certainty over the entire forecast horizon. The job of providing the legislative decision-makers with a probability distribution of tax revenues might then seem rather easy. Take the various values of *ngnp* generated by the various scenarios, calculate the state tax revenues for each scenario, and then assign the subjective probability for that scenario to the corresponding tax revenue estimate. Although this use of the scenarios and subjective probabilities might seem intuitively appealing, it is almost certainly wrong.

Chart 3 helps illustrate why this interpretation is wrong. Its vertical and horizontal axes measure *cpi*, the 1982–85 average annual percentage change in the consumer price index (CPI), and *rgnp*, the 1982–85 average annual percentage change in real GNP. The sum of these two numbers is approximately *ngnp*, which is a major ingredient in Minnesota's tax revenue forecast. The points labeled c (Control), o (Optimistic), s (Stagflation), and w (W-Recession—that is, one recession soon after another, a w-shaped path) describe the actual scenarios for these variables that were generated by the consulting service for Minnesota's Finance Department to use in preparing its January 1983 revenue forecast for the 1984–85 biennium. The consulting service assigned subjective probabilities of

Chart 3

Regions assigned to the consulting service's scenarios cannot be expected to contain all the values of *ngnp* that generate the same revenue.



50 percent, 10 percent, 15 percent, and 25 percent, respectively, to these scenarios but did not specify the ranges around the scenarios to which these subjective probabilities applied. This imprecision considerably limits the usefulness of the subjective probability statements. However, since we need the ranges to complete this example, we will arbitrarily divide all possible *rgnp-cpi* outcomes into the four regions shown in Chart 3 and will proceed as though these were the ranges the consulting service had in mind.

Even given these ranges, however, it is still not possible to translate the scenario into revenue estimates. For example, because scenario w has a subjective probability of 25 percent, region W is assigned the same probability. But observe the difficulty this interpretation would create for the Finance Department. Assuming *t* is known and equal to zero, the Finance Department would report that scenario w generates an *ngnp* of 5.6 percent, the weakest of the consulting service's four scenarios. Consequently, if the legislature follows the intuitive interpretation of subjective probabilities, it is led to believe that the probability that revenues (*tr*) will grow less than 5.6 percent is on the order of 25 percent. But this interpretation is, in general, simply wrong. In Chart 3, points ws, wc, and wo all generate the same 5.6 percent increase in *ngnp* and, hence, the same tax revenues. These points, however, all lie in regions other than W. Consequently, the probability statements about Minnesota revenues growing at less than the 5.6 percent rate associated with w are almost certainly wrong because region W contains many possible outcomes in which Minnesota revenues would grow faster than 5.6 percent and omits many others in which revenues would grow slower than 5.6 percent. Furthermore, with only four scenarios and four large regions, there is no assurance that these two sources of error even approximately cancel. Thus, when the state legislature makes budget decisions on the basis of revenue estimates generated by scenario w, it has no scientific basis on which to evaluate the probability of this outcome or any deviation therefrom.

Even if the subjective probabilities the consulting service assigns to individual scenarios could be used as subjective probabilities for *ngnp* in estimating Minnesota's revenues,[5] they would still give a misleading picture of the true probabilities of possible Minnesota revenue outcomes. There are at least two reasons for this assertion.

First of all, the description of the range of possible revenue outcomes lacks detail. There is, for example, no way to respond to such questions as, What is the probability that *tr* will be 1 percent below the growth in revenue generated by scenario w or 1 percent above the growth in revenues generated by scenario o? The consulting service simply provides no information about these events, and yet legislative fiscal contingency planning must surely depend on their probabilities.

Perhaps an even more important reason is that *t* is not known to the Finance Department with certainty. Even if there were no changes in the tax law and if the economy were to evolve precisely according to one of the consulting service's scenarios, the factors that determine Minnesota's effective tax rate are so complex that errors in forecasting Minnesota revenues would frequently occur. Because *t* is not known with certainty, the range of revenue outcomes will, in general, be different from the range implied by the consulting service's high and low scenarios, and the Finance Department has no way of determining even the sign of the difference, let alone the magnitude. If the effective tax rate is positively correlated with *ngnp*, the revenue range implied by the consulting service's scenarios is too narrow; but if there is a negative correlation, the range is overstated.

We have noted several difficulties in the current procedure for translating the scenarios and probability statements that consulting services provide for the national economy into objective measures of the uncertainties in Minnesota's revenue forecasts. The probability statements primarily reflect subjective judgments rather than accepted statistical procedures. The scenarios and subjective probabilities also lack the detail and precision needed for a valid translation. Finally, the current procedure ignores the random nature of Minnesota's effective tax rate. In light of these observations, we conclude that Minnesota's Finance Department should reduce its emphasis on the consulting service's subjective probabilities of national scenarios and focus its efforts on generating probability distributions of something more relevant to the state's fiscal contingency planning, namely, state tax receipts.

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[5] The only way we can establish any scientific basis for the subjective probabilities is to divide the *rgnp-cpi* possibilities among the four scenarios so that the lines dividing the regions are parallel to the constant *ngnp* lines. In our description of a value-based tax system, this choice of regions is special because it alone assigns all national scenarios that generate the same state revenues to a single region with an attached subjective probability. We doubt, however, that consulting services have these sorts of regions in mind when they state their subjective probabilities. In Chart 3, for example, the consulting service's scenarios c and o generate almost the same *ngnp*, so separating them by a constant *ngnp* line seems somewhat pointless and implausible.

Table 2

VAR predicts Minnesota tax revenues and measures the uncertainty in the forecasts.

Forecasts Made in January 1983 Based upon Actual Data through the Fourth Quarter 1982

	Individual Income Tax		Corporate Income Tax		Sales Tax		Total	
Fiscal Year	$ Millions	Standard Error*	$ Millions	Standard Error	$ Millions	Standard Error	$ Millions	Standard Error
1983	1,883	3.8%	245	6.6%	904	3.2%	3,032	2.5%
1984	2,236	8.3	325	14.8	982	9.2	3,544	6.3
1984–85 Biennium	4,905	7.6	710	13.4	2,041	9.3	7,657	6.2

*One standard deviation from a sample of 1,000 simulations.

VAR Can Measure Uncertainty

VAR can provide more objective estimates of the uncertainty in the revenue forecast than the current procedure can, thus enabling the state to better plan for shortfalls. VAR can assign probabilities to the various revenue forecasts because an objective and more complete probability specification than the current procedure provides is inherently part of the procedure. The probability specification is objective because it is derived from the actual history of forecast errors that are generated by the procedure over the entire sample period. When the forecaster looks at the future, these past errors provide objective estimates of the probability distribution of errors that can be expected to occur when the procedure is consistently used to forecast the future.

Such probability distributions can be derived for any subset of the variables in the model. In particular, they can be calculated for the tax revenue variables. For this reason, the VAR procedure produces a more complete specification of the probability distribution of tax revenues than Minnesota's current procedure because VAR translates, in a quantitative way, the uncertainty in the predictions of the national variables into the uncertainty in the predictions of the variables of direct concern, namely, the tax revenue estimates.

One quantification of the uncertainty in the revenue forecasts is given by the standard errors of the forecast generated by the forecasting procedure itself. Table 2 shows such standard errors generated by our VAR procedure for forecasts produced in January 1983 (at the time of the governor's budget message) for fiscal years 1983 and 1984 and for the 1984–85 biennium.[6] According to our calculations (as shown in Table 2), the expected receipts from the three major revenue sources for the 1984–85 biennium were about $7,657 million as of January. This estimate is not significantly different from the estimates produced by the Finance Department for the governor's budget proposal. The forecast error in our estimate is 6.2 percent, which means that there is about a 67 percent chance that revenues will fall within a range of $474 million of the mean. Alternatively, if the legislature were to set expected revenues at $7,657 million, in January we would have estimated that there was a 17 percent chance that there would be a revenue shortfall of at least $474 million.[7]

Another, and more complete quantification of the uncertainty in the revenue forecast generated by our VAR procedure is an estimate of the entire probability distribu-

[6] The forecast errors for the separate tax categories vary rather widely and are all higher than the forecast error for total tax receipts. Even though total tax receipts are simply the sum of the three separate taxes, the variation in the total is not a simple average of the variation of the separate taxes because of the correlations.

[7] These probability statements (as well as those described below) are generated by the VAR procedure itself via the statistical technique known as Monte Carlo simulation. The historical pattern of errors is used to estimate a probability distribution of the forecast errors of the VAR procedure. This distribution considers not only each variable alone but also the correlation of the forecast errors across the several variables in the model. For each quarter of the forecast horizon, a sample value of the error for each variable in the model is drawn from the estimated distribution. Thus, a sample path is generated for each variable in the model, and this process is repeated 1,000 times. The probability statements then represent the proportion of sample paths that satisfy certain criteria. For example, in Table 2, about 170 of the 1,000 sample paths had biennial revenues of $7,183 million or less.

Chart 4

VAR's prediction of Minnesota tax revenues for the 1984–85 biennium approximates a normal probability distribution.

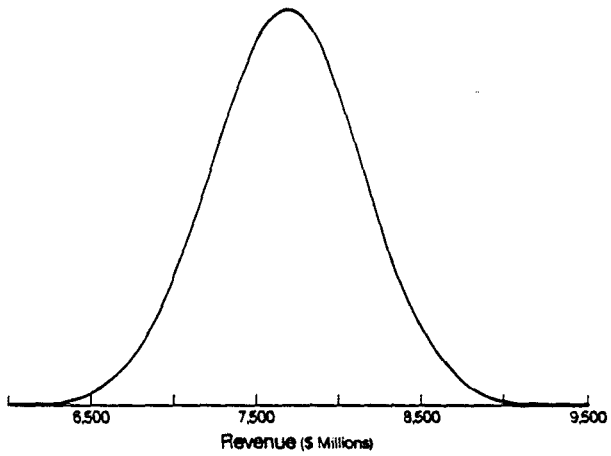
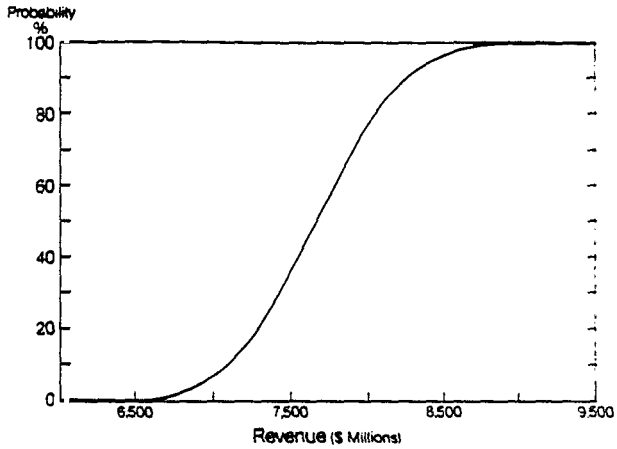


Chart 5

That distribution can also be depicted as the probability that revenues will be less than or equal to a given level. . .



tion of possible revenues. Such a quantification is complete because it contains all the information about the future that the forecasting procedure can generate.

Because the distribution of tax revenues in the VAR sample can be reasonably approximated by a normal distribution, we have shown this traditional "bell-shaped" representation of revenues for the 1984–85 biennium in Chart 4. A possibly more convenient representation of the same information is shown in Chart 5. For any particular level of revenues shown on the horizontal axis, the chart quantifies the probability that actual revenues will be less than or equal to that amount.[8] This chart provides complete and objective measures of the uncertainty in our VAR's forecasts of Minnesota's revenues.

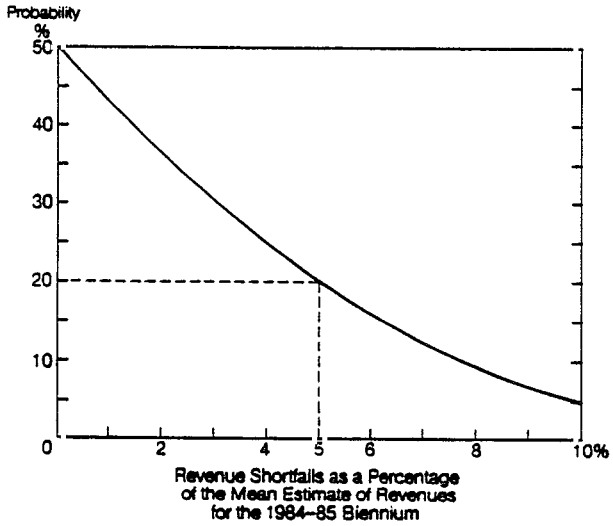
The probability distribution of revenue possibilities can be used, for example, for choosing the level of a reserve fund. As we explained above, it makes no sense to choose a reserve fund that is so large that it insures against events that have little probability of occurring. The decision-makers must weigh the costs of setting the reserve fund at a given size against the risk that a revenue shortfall will exhaust the fund. Thus, the decision process requires that the chances of shortfalls of various sizes be objectively measured. The VAR procedure can generate such a quantification from the information contained in Chart 5, and this is illustrated in Chart 6. Revenue shortfalls are shown on the horizontal axis as a percentage of the mean, and the probability shown on the vertical axis is an estimate of the probability that a shortfall of at least the size shown on the horizontal axis will occur.

To understand how Chart 6 can help in establishing a reserve fund, suppose the Minnesota legislature had accepted for planning purposes the VAR mean estimate of revenues for the 1984–85 biennium shown in Table 2, and suppose that it had established a spending budget equal to this mean with 5 percent allocated to a reserve fund. Then Chart 6 would tell the legislature that there is a 20 percent chance (about once in five biennia on average) that a reserve fund of this size would be inadequate to cover the actual revenue shortfall. If the reserve fund were doubled to 10 percent of expected revenues, then the probability of a shortfall in excess of the reserve fund would be reduced to about 5 percent. Thus, Chart 6 provides an objective estimate of the risk attached to each level of the reserve fund

[8] Note that if the VAR and the state procedures are combined to produce a more accurate forecast, the quantification of uncertainty described here is not exactly correct because an element of judgment is brought into the procedure. Because the differences in the VAR and combined values, as shown in Charts 1 and 2, are rather small, we proceed as if the VAR measure of uncertainty were still applicable.

Chart 6

. . .and that probability enables VAR to quantify the chance that a shortfall of a given size will occur.



that the state might choose and is therefore an important element in the reserve fund decision process.[9]

Conclusion

Adequate fiscal contingency planning requires not only a good estimate of the expected value of revenues but also an objective estimate of the probable errors in the forecast. The latter point may be even more important than the former because even the best currently known forecasting procedures are subject to a wide range of errors. The VAR procedure that we have described provides the objective quantification of forecast errors that is missing from Minnesota's current procedure while forecasting, on average, at least as accurately and at a much lower cost. For these reasons, we propose that Minnesota decisionmakers incorporate VAR into their forecasting procedures. Other economic decisionmakers may also wish to consider incorporating VAR, or similar procedures, into their contingency planning.

[9]By using the reserve fund to illustrate the value of the information the VAR procedure provides, we in no way wish to imply that, in our judgment, establishing a reserve fund is the best way to plan for revenue shortfalls. Our point is simply that whatever method is used, the information contained in Chart 6 is relevant to the decision process.

Appendix
Improving VAR's Accuracy with Additional Data

Additional information improves VAR's performance markedly. Table A1 reports the mean absolute errors over four different horizons for our VAR procedure utilizing four different sets of information. We based the first forecast on information available through the fourth quarter of the previous year (we call this the January forecast) and made no specific attempt to incorporate information about legislated changes in the tax code into the forecast procedure. The other three forecast sets differ either with respect to timing (the April forecast assumes that first-quarter data is available) or with respect to the specific incorporation of legislated tax changes into the forecast. By adding another quarter of actual data and information about legislative tax law adjustments, we reduced the average forecast error for total tax receipts over the following biennium from 11.9 to 5.4 percent.

Our method of incorporating legislated tax law changes into our VAR forecast was very crude but, as Table A1 shows, rather effective. However, we have little doubt that a more

Table A1

VAR's forecasts improve with more information.

Errors in 1971–82 Minnesota Forecasts Made in January and April without and with Tax Law Adjustments*

	Fiscal Year		Biennium	
	Current	Following	Current	Following
Without				
January				
Personal Income	1.0	3.3	2.1	4.1
Individual Income Tax (net)	4.0	9.7	6.2	12.8
Total Tax Receipts (net)	2.7	8.4	5.3	11.9
April				
Personal Income	0.3	2.5	1.4	3.3
Individual Income Tax (net)	2.0	8.6	4.9	10.7
Total Tax Receipts (net)	1.3	6.6	3.7	8.9
With				
January				
Personal Income	1.0	3.3	2.1	4.1
Individual Income Tax (net)	3.7	5.7	3.9	8.1
Total Tax Receipts (net)	2.1	5.7	3.4	8.5
April				
Personal Income	0.3	2.5	1.4	3.3
Individual Income Tax (net)	1.8	5.1	3.1	5.5
Total Tax Receipts (net)	1.2	4.3	2.7	5.4

*Mean absolute value of the following: the difference between the forecasted and actual levels as a percentage of the actual level.

detailed examination of Minnesota's tax law could produce a better measure than ours.

We considered only tax law changes beginning in late 1967 after the initial setting of Minnesota's sales tax. We then considered only changes in the tax law whose effects on revenues had been estimated by the Revenue Department. (These estimates were made essentially at the time the law was enacted so they were made before there was any information on the actual impact of the tax law change.) Except for translating dollar magnitudes into percentages, we made no independent estimates of the fiscal impact of tax legislation; and, except for a few obvious cases, we made no attempt to allocate tax changes within a fiscal or calendar year. Our primary data source was a set of documents of legislative actions that the Revenue Department kindly provided to us. The staff of the Revenue Department was not aware of our methodology in constructing the tax law adjustment until after all the calculations in this study were completed, so they should not be held responsible for any possible misuse of the fiscal impact data that we might have made in this report.

Because all three tax variables entered our model in logarithmic form, it was necessary to translate all fiscal actions into percentage terms. The three tax law adjustment variables that were used in our VAR procedure are shown in Table A2 where the units are such that the first difference defines the percentage change in revenues due to the change in the tax law.

Table A2

Values of the Tax Law Adjustment Variables*

Quarter of Calendar Year		Individual Income Tax	Corporate Income Tax	Sales Tax
1967	Third	0	0	0
	Fourth	0	0	0
1968	First	0	0	0
	Second	0	0	0
	Third	0	0	0
	Fourth	0	0	0
1969	First	0	0	0
	Second	0	0	0
	Third	0	0	0
	Fourth	0	0	0
1970	First	0	0	0
	Second	0	0	0
	Third	0	0	0
	Fourth	0	0	0
1971	First	0	0	0
	Second	0	0	0
	Third	0	0	0
	Fourth	19.9	47.6	15.1
1972	First	19.9	47.6	45.3
	Second	19.9	47.6	26.4
	Third	19.9	47.6	26.4
	Fourth	19.9	47.6	26.4
1973	First	19.9	47.6	26.4
	Second	19.9	47.6	26.4
	Third	19.9	47.6	26.4
	Fourth	19.9	47.6	26.4
1974	First	19.9	47.6	26.4
	Second	19.9	47.6	26.4
	Third	19.9	47.6	26.4
	Fourth	19.9	47.6	26.4
1975	First	19.9	47.6	26.4
1975	Second	19.9	47.6	26.4
	Third	18.4	47.6	26.4
	Fourth	18.4	47.6	26.4
1976	First	18.4	47.6	26.4
	Second	18.4	47.6	26.4
	Third	18.4	47.6	26.4
	Fourth	18.4	47.6	26.4
1977	First	18.4	47.6	26.4
	Second	18.4	47.6	26.4
	Third	18.4	47.6	26.4
	Fourth	18.4	47.6	26.4
1978	First	12.5	47.6	26.4
	Second	12.5	47.6	41.0
	Third	12.5	47.6	41.0
	Fourth	12.5	47.6	41.0
1979	First	12.5	47.6	41.0
	Second	12.5	47.6	41.0
	Third	-5.0	47.6	41.0
	Fourth	-5.0	47.6	41.0
1980	First	-5.0	47.6	41.0
	Second	-5.0	47.6	41.0
	Third	-5.8	47.6	41.0
	Fourth	-5.8	47.6	41.0
1981	First	-5.8	47.6	41.0
	Second	11.8	47.6	41.0
	Third	-0.1	47.6	64.4
	Fourth	-0.1	47.6	64.4
1982	First	6.7	56.9	64.4
	Second	6.7	56.9	64.4
	Third	8.8	58.2	66.7
	Fourth	8.8	58.2	66.7

*These variables estimate the percentage difference between Minnesota's actual post-1967 taxes and what those taxes would have been if Minnesota's 1967 tax laws had still been in effect. These estimates were constructed from Revenue Department estimates of the effect (in dollars) of each tax law change on tax revenues in the fiscal year (or biennium) that tax law change took effect. To get the values shown above, the change in revenue estimated by the Revenue Department for a given tax law change was first expressed as a percentage of actual revenue in the fiscal year (or biennium) that tax law change took effect. This percentage was then assumed to give the effect of that tax law change on revenues in all subsequent quarters. Each quarterly figure above gives the cumulative percentages for all tax law changes in effect up to that point.

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