Federal Reserve Bank of Minneapolis Quarterly Review
Vol. 21, No. 1, Winter 1997, pp. 2–20

Absence-of-Double-Coincidence Models of Money: A Progress Report

Neil Wallace
Adviser
Research Department
Federal Reserve Bank of Minneapolis
and Barnett Banks Professor of Money and Banking
University of Miami

Abstract

This study describes a model built on the long-held view that the use of money as a medium of exchange is the result of an absence of double coincidence of wants. The model can account for two of the most challenging observations facing monetary theory: the disparate short-run and long-run effects of changes in the quantity of money and the coexistence of money and assets with higher rates of return. For both observations, the model's ability to provide a rich analysis depends on little more than the ingredients implicit in the absence-of-double-coincidence view.

In 1968, Samuelson (1968, p. 171) described the unsatisfactory state of monetary theory by describing the experience of economics professors moving from one class period to the next:

> In a real sense there *was* a dichotomy in our minds; we were schizophrenics. From 9 to 9.50 a.m. we presented a simple quantity theory of neutral money. There were then barely ten minutes to clear our palates for the 10 to 10.50 discussion of how an engineered increase in M [the amount of money] would help the economy.

The paper in which this description appears demonstrates that the neutrality of money, a conclusion of an incoherent model, also holds in a less incoherent model. The incoherent model is the Arrow-Debreu model of general competitive equilibrium with a quantity theory equation attached to it.[1] That model is what was taught from 9 to 9:50. The less incoherent model is the Arrow-Debreu model of general competitive equilibrium with real money balances as an additional argument of utility functions. That model is not what was taught from 10 to 10:50, and Samuelson's paper is not about remedying the schizophrenia he talks about in the above passage. Instead, Samuelson's paper is intended to ameliorate a more fundamental schizophrenia concerning the way economists think about money.

The *money* in the quantity theory equation or in Samuelson's utility function is, no doubt, meant to be a medium of exchange, a means of accomplishing exchanges between people.[2] The use of a medium of exchange has long been believed to be the result of real frictions that give rise to what has come to be called an *absence of double coincidence of wants.* The more fundamental schizophrenia afflicting the economics profession is that while holding that belief, the great majority of economists working on monetary issues have been and are using models that somehow shove money into the frictionless setting that forms the background for the Arrow-Debreu model of general competitive equilibrium. Both of the models Samuelson discusses, the quantity theory model and his utility function model, are examples of that sort of model.

Perhaps the earliest statement about the role of money in overcoming the lack of double coincidences is from the 2nd–3rd century Roman jurist Paulus who, according to Monroe (1966, p. 10), said, "Since occasions where two persons can just satisfy each other's desires are rarely met, a material was chosen to serve as a general medium of exchange." The continued belief in that role of money is evidenced by the prominence given to it ever since. For example, Smith (1776, pp. 22, 23) uses a version of it to open the chapter of *The Wealth of Nations* titled "Of the Origin and Use of Money":

> When the division of labour has been once thoroughly established, it is but a very small part of a man's wants which the produce of his own labour can supply. He supplies the far greater part of them by exchanging . . . .
>
> But when the division of labour first began to take place, this power of exchanging must frequently have been very much clogged and embarrassed in its operations. One man, we shall suppose, has more of a certain commodity than he himself has occasion for, while another has less. The former consequently would be glad to dispose of, and the latter to purchase, a part of this superfluity. But if this latter should chance to have nothing that the former stands in need of, no exchange can be made between them . . . . In order to avoid the inconveniency of such situations, every prudent man in every period of society, after the first establishment of the division of labour, must naturally have endeavoured to manage his affairs in such a manner, as to have at all times by him, besides the peculiar produce of his own industry, a certain quantity of some one commodity or other, such as he imagined few people would be likely to refuse in exchange for the produce of their industry.

And the belief appears in Mishkin's 1986 (p. 22) textbook as motivation for the existence of money: "In a barter economy, if Ellen [who can produce only brilliant economics lectures] wants to eat, she must find a farmer who not only produces the food she likes, but also wants to learn economics."

Consistent with economists' schizophrenic view of money, until very recently no one has actually used the absence-of-double-coincidence idea in any work on money. Thus, for example, Smith (1776) makes no use of it in the rest of *The Wealth of Nations,* and Mishkin (1986) makes no use of it in the rest of his text. The first attempt to use or build on the absence-of-double-coincidence idea, as opposed to paying lip service to it, appeared in Ostroy 1970. And not until the 1980s, in Diamond's (1984) work on search models and in the subsequent work of Kiyotaki and Wright (1989), did economists succeed in constructing coherent models built on the absence-of-double-coincidence idea. Kiyotaki and Wright's (1989) model is especially noteworthy because theirs is the first model in which several objects are potential media of exchange and in which the relationship between the physical properties of those objects and their role as media of exchange can be studied.

Although models built on the absence-of-double-coincidence idea provide a remedy for economists' schizophrenia concerning money, considerable skepticism remains about the value of those models. A primary concern is whether such models are able to account for two observations that have been regarded as the main challenges for monetary economics. One observation is the disparate long-run and short-run effects of changes in the quantity of money: the seeming tendency for long-run effects to be primarily nominal and short-run effects to be predominantly real, the disparity which is the source of the schizophrenia Samuelson talks about. The other observation is described by Hicks in a famous 1935 paper as the greatest challenge facing monetary theory: the coexistence of money and assets with higher rates of return. Why is money held when higher-return assets are available?

Here I describe a particular model built on the absence-of-double-coincidence idea and demonstrate that that model can account for both of the challenging observations about money. That demonstration constitutes the main part of this progress report. In addition, I deal briefly with another concern about existing models that are built on the absence-of-double-coincidence idea: their extreme features. I indicate how the particular model I use, which is extreme in a number of respects, can be generalized so that the real frictions can range from being present in extreme ways to being absent. In the limiting case in which they are absent, the model resembles the frictionless setting of the Arrow-Debreu model of general competitive equilibrium.

An Absence-of-Double-Coincidence Model

I start by describing, in detail, a particular model which incorporates the absence-of-double-coincidence idea. To emphasize the connections between the model and long-held views about the role of a lack of double coincidences, I use the Smith (1776) passage above to motivate the structure of the model. First, the model contains more than one time period; otherwise, Smith's remark about what "few people would be likely to refuse" cannot play a role. Second, the model has two-person meetings, because Smith's discussion is in terms of such meetings. In particular, Smith's discussion is inconsistent with meetings of everyone, which is the explicit or implicit assumption in most models with markets. Third, a specialization pattern among people motivates trade and is consistent with the lack of double coincidences in two-person meetings. Finally, and less evident, something in the model prevents trade from being accomplished through some prior arrangement or through the use of some form of credit, for example, IOUs.[3]

In regard to time periods, I assume that time is discrete and continues indefinitely, because a certain, or sure, last date creates problems, especially if money consists of intrinsically worthless objects like stones, shells, or pieces of paper that are not promises to be redeemed for anything else. Thus, I let the current date be date 0, and I let subsequent dates, in order, correspond to the positive integers 1, 2, 3, and so on.

Next, I assume a specialization pattern that implies a complete lack of double coincidences in meetings between two people. Let there be N perishable goods at each date, where N is an integer no smaller than 3. By *perishable,* I mean that those goods have to be consumed in the same period they are produced, or they are lost. Each person in the model is one of N types, where type is determined by the following specialization pattern in consumption and production among the N goods: a type n person wants to consume only good n and can produce only good $n + 1$ (modulo N, so that type N produces good 1), for $n = 1, 2, ..., N$. Thus, with $N \geq 3$, when any two people meet, there cannot be a double coincidence. There can be no coincidence, which means that neither person produces what the other consumes, or there can be a single coincidence, which means that one person produces what the other consumes but not vice versa. A single-coincidence meeting occurs when and only when a type n person meets a type $n + 1$ (modulo N) person for some integer n from 1 to N.

Further, in regard to preferences, a type n person likes consuming (gets utility from consuming good n at any date) and dislikes producing (gets disutility from producing good $n + 1$ at any date). Total utility, or well-being, at any date is given by $u(x) - y$, where x is the amount of good consumed and y is the amount of good produced.[4] The function u is defined on $[0,\infty)$, is increasing, is unbounded above, is twice differentiable, and satisfies $u(0) = 0$, $u'' < 0$, $u'(\infty) = 0$, and $u'(0) = \infty$. (Both the utility of consuming, the function u, and the disutility of producing, the identity function, are shown in the accompanying figure. An example of the u function which satisfies all the assumptions is $u(x) = x^{1/2}$.)

Each person lives throughout the life of the economy. Therefore, I have to describe how the person weights well-being across dates. I assume that the person discounts well-being in the future at a constant rate, denoted β, where β is strictly between zero and one. Finally, since there is uncertainty, I assume that each person acts so as to maximize expected discounted utility. Thus, if E denotes the expected value at the start of date 0, then a type i person acts to maximize $E\Sigma_{t=0}^{\infty}\beta^t[u(x_t) - y_t]$, where x_t denotes the amount consumed (good n for a type n person) at date t and y_t denotes the amount produced (good $n + 1$ for a type n person) at date t. Notice that u and β, the determinants of preferences, are independent of type. That is one of several symmetries over types I assume to keep the model simple.

Another symmetry assumption is that there are equal numbers of each type of person. In particular, there is a [0,1] continuum of each type, meaning that there is a person of each type corresponding to each real number between 0 and 1.

Now I describe how meetings occur in the model. I want to end up with two-person meetings. If I give people in the model too much freedom to choose whom they meet, then I will not end up with two-person meetings. In the model as so far specified, the only reason for people to meet is to produce and trade, and the larger the number of people in a meeting, the richer the production and trade possibilities. Therefore, to end up with two-person meetings, I have to assume that for more than two people to meet is difficult or costly. In effect, I assume that it is impossible or infinitely costly. I also assume that people cannot choose whom to meet. Instead, each person at each date is paired at random with one other person, where *at random* means that the probability of one person meeting another person with particular characteristics is the fraction of those particular people in the entire population. Therefore, the fraction of all meetings that are single-coincidence meetings is $2/N$. In summary, people do not choose whom they meet in this model. At each date, each person runs into someone at random. In other words, this random meeting is free, and any other meeting at that date is impossible, is infinitely costly.

In addition, people cannot commit themselves to future actions. If they could—if at each meeting, some outside enforcement could punish the participants for not carrying out some explicit or implicit promise they made earlier—then each producer could commit to producing y^* in a single-coincidence meeting, where y^* is the unique solution to $u'(y^*) = 1$. (See the figure.) That would be a good outcome and, given the enforcement, does not require the use of a medium of exchange. I assume that such commitment is not possible.

Finally, it may seem as if the assumption of pairwise meetings with a continuum of people rules out the use of credit. This assumption does rule out the possibility that an IOU issued by a person gets back to the issuer with positive probability. Therefore, such IOUs cannot be valuable, because if they were, everyone would always issue them in order to consume and never produce. However, ruling out such IOUs is not enough. I also need to rule out the following golden-rule form of credit. If a producer in a single-coincidence meeting thinks that he or she will receive a gift in future single-coincidence meetings if he or she gives a gift today, then the producer may well give the gift today. To rule that out, I assume that each person's history—what he or she did in meetings in the past—is known only to the person, except to the extent that the history is revealed by what the person owns now.[5] That assumption implies that a person who fails to give a gift cannot be punished by not being given gifts subsequently, because no one in the future knows whether the person gave a gift in the past.

Although the model as so far specified is special and extreme, nothing in it is very distant from what is in Smith 1776. The model is special and extreme to make it simple. That simplicity will be appreciated later. While Smith had in mind that people choose to specialize in production and, perhaps, to an extent, in consumption, I simply assume that technologies and preferences are specialized. Also, Smith might view the random meetings as extreme. But, as noted above, that is a simple way to get the two-person meetings he was talking about. It may seem, however, that I have gone too far in the direction of clogging trade. Since there are no double coincidences, since goods are perishable, and since credit of any sort is impossible—can anything happen when two people meet? Nothing can happen unless some type of storable asset is put into the model. I do that in the two sections that follow.

In the next section, in which I take up the long-run vs. short-run challenge, I introduce one kind of asset, an intrinsically useless object I call a *fiat asset.* The fiat asset functions as money in the model, and my analysis focuses on the effects of changes in its quantity. In the succeeding section, in which I take up the coexistence challenge, I introduce two assets—a fiat asset and a *dividend-bearing asset*—the quantities of which never change. Despite those differences, there are some common features in the two sections. The N goods described above are perfectly divisible, so that a producer is able to produce any nonnegative amount. In contrast, the assets I introduce below are indivisible: they come in integer units. Also, in contrast to the goods, they can be stored from one date to the next indefinitely. However, throughout I assume that each person can store from one date to the next, at most, one unit of some asset. The only rationale for this extreme assumption is that it makes results relatively easy to get. Because this assumption has no other rationale and because it is so extreme when coupled with the assumption that assets are indivisible, I comment in detail on what I think are the consequences of weakening it by, for example, letting people store any amount of assets.

With assets in the model, there is a possibility for trade. Exactly how that trade occurs is described in the sections that follow. Trade is somewhat different when the model has one asset than when it has two. Essentially, trade occurs if consumers in single-coincidence meetings have assets that producers "imagined few people would be likely to refuse [in the future] in exchange for the produce of their industry" (Smith 1776, p. 23).

The Long-Run vs. Short-Run Challenge

Changes in the quantity of money are often accompanied by disparate long- and short-run effects. In particular, the long-run effects are often predominantly nominal, with the price level changing in the same direction as the change in the quantity of money, while the short-run effects are often predominantly real, with output and employment changing in the same direction as the change in the quantity of money. It is not an exaggeration to say that most of macroeconomics, at least before the 1980s, has been an attempt to explain these disparate effects. Certainly, this is true of Keynes'(1936) *General Theory* and the work for which Lucas was awarded the 1995 Nobel prize. Commenting on his work, Lucas (1996, p. 661) says,

> The work for which I have received the Nobel Prize was part of an effort to understand how changes in the conduct of monetary policy can influence inflation, employment, and production. So much thought has been devoted to this question and so much evidence is available that one might reasonably assume that it had been solved long ago. But this is not the case: It had not been solved in the 1970s when I began my work on it, and even now this question has not been given anything like a fully satisfactory answer.

Lucas' comment is enough to establish the sense in which the observation of disparate long- and short-run effects of changes in the quantity of money is challenging. The observation is also important: how one accounts for the observation will play a crucial role in determining one's views about the desirable time paths of the quantity of money.

The absence-of-double-coincidence model can produce those disparate long- and short-run effects, provided two main requirements are met: (1) The quantity of money is random. (2) People learn what happened to the quantity of money with a lag. These requirements are not new; they are important ingredients in several models consistent with the observed long- and short-run effects of changes in the quantity of money, including Lucas' Nobel prize–winning work. That being so, one may reasonably ask, What is gained by demonstrating that those requirements give rise to similar effects in an absence-of-double-coincidence model? The answer is that doing so shows that the ingredients in the model, ingredients which give money a role in overcoming the lack of double coincidences, are sufficient to account for those effects.

In addition, the absence-of-double-coincidence model suggests a new perspective on requirement (2), which is often regarded as implausible for a modern economy. I noted above that restrictions on what people know about others are necessary to prevent some form of credit from overcoming the lack of double coincidences. Against the background of such restrictions, which seem plausible even in a modern economy, requirement (2), which also limits what people know, ought to seem less implausible.

The Fiat Asset

I assume here that the only asset is the fiat asset. Hence, if anything is to play a monetary role, it is the fiat asset. As noted above, the fiat asset is indivisible, and each person can store, at most, one unit of it. At the start of date 0, there exists some quantity of the fiat asset per type, a quantity denoted m_0, which I assume to be positive and less than one. Notice that if the entire quantity of the fiat asset is held and if, as I assume, the initial holdings are distributed symmetrically across types of people, then the fraction of each type of person holding one unit is m_0 and the fraction of each type holding nothing is $1 - m_0$.

Changes in the quantity of the fiat asset, or money, come about as follows. At the end of date 0, there is a one-time increase in the aggregate quantity of it. This increase, which itself is random, enters the economy through a discovery process that is random among the people who leave meetings at date 0 with nothing. In particular, at the end of date 0, there is a once-and-for-all increase in the amount of the fiat asset. This increase per type, denoted Δ, is a drawing from the following distribution, which is common knowledge at the start of date 0: $\Delta = \Delta_k$ with probability p_k, $k = 1, 2, ..., K$, where $p_k > 0$, $K \geq 2$, $\Delta_{k+1} > \Delta_k$, Δ_1

> 0, $m_0 + \Delta_K \leq 1/2$, and the range of Δ, $\Delta_K - \Delta_1$, is sufficiently small for reasons to be described later. Conditional on Δ, each person who leaves a date 0 meeting without the fiat asset discovers a unit of it with probability $\Delta/(1-m_0)$. This specification satisfies requirement (1), randomness of changes in the quantity of money. To satisfy requirement (2), I assume that at date 1, no one knows Δ, although people use their experience to help them decide what happened to the quantity of the fiat asset. That is, they use their experience regarding discovery and their experience regarding whether the person they meet at date 1 has the fiat asset. Finally, I assume that everyone learns the realization of Δ at the start of date 2.

I study a once-and-for-all change in the quantity of the fiat asset because it is simple. I assume that only those who leave meetings without the fiat asset are eligible to discover a unit of it, because those with a unit would have to discard it if they discovered an additional unit.[6] The assumption that $m_0 + \Delta \leq 1/2$ restricts the quantity of the fiat asset to a range in which the probability of a meeting occurring in which one person has the asset and the other does not, a necessary condition for trade to occur in this model, is increasing in its quantity. Since this necessary condition arises only because of the upper bound on individual holdings and since the upper bound is adopted only for tractability, it seems sensible to restrict the quantity of the fiat asset to a range such that the probability of a meeting occurring between someone with the asset and someone without it is increasing in the quantity. That range is [0,1/2), because if m is the quantity of the fiat asset per type, then the fraction of all single-coincidence meetings in which the potential consumer has it and the potential producer does not is $(1-m)m$, which is increasing in m for $m < 1/2$. Finally, the assumption that everyone learns the realization of Δ at the start of date 2 is also made for simplicity. It allows me to describe what happens at dates 0 and 1 by working backward from a simple description of what happens at date 2.

The Equilibrium Concept

An *equilibrium* is a description for each date, starting at date 0 and stretching into the indefinite future, of the frequencies with which different kinds of meetings occur and of what happens in those meetings. The sequence of actions at each date is as follows. At the start of each date, each person has either one unit of the fiat asset or nothing. Then people meet in pairs at random. Because of the indivisibility of the asset and the upper bound on individual holdings, there is a potential for trade only when a type n person meets a type $n + 1$ person and the type $n + 1$ person, the potential consumer, has the fiat asset and the type n person, the potential producer, does not. I call such meetings *trade meetings.* People in trade meetings bargain. If the outcome of bargaining implies exchange, then production and consumption occur. At all dates other than date 0, that is all that happens. At all such dates, people take what they left the meeting with, either a unit of the fiat asset or nothing, and start the next date. Date 0 is slightly different. After the meeting at date 0, each person who leaves a meeting with no fiat asset has a chance of randomly discovering a unit of it. Those who do discover a unit start date 1 with the fiat asset.

In a trade meeting, the following simple bargaining rule is assumed: The potential consumer makes a take-it-or-leave-it offer consisting of a demand for some amount of production in exchange for the consumer's asset, and the potential producer accepts if made no worse off by accepting. A consequence is that the consumer makes so stringent a demand on the producer, demands so much production, that the producer is just on the margin between accepting the demand and rejecting it. I assume, as is standard, that in this circumstance, the producer accepts. The producer is willing to produce at all only because the producer thinks he or she will be able to use the acquired fiat asset in a subsequent trade meeting in which he or she is the consumer. Hence, the producer's view about the future is crucial. As part of the equilibrium concept, I assume, as is now standard, that the producer has rational expectations about his or her subsequent potential uses of the fiat asset. The above take-it-or-leave-it bargaining and rational expectations constitute the equilibrium concept.

Such take-it-or-leave-it bargaining and rational expectations do not completely pin down what happens. I focus on only one of the things that can happen—a symmetric equilibrium in which the fiat asset has a positive value which is constant from date 2 onward. I do not discuss potential equilibria which are not symmetric and constant starting at date 2. Not much is known about such equilibria. I also do not discuss an equilibrium in which the fiat asset does not have value. At the cost of slightly complicating the model, that equilibrium could be eliminated.[7]

As noted above, the assumptions permit existence of equilibria which are symmetric across types of people. One of those assumptions is that the initial asset distribution is symmetric across types. It is easy to show that if the asset distribution at the start of a date is symmetric and if trades and discoveries are symmetric, then the asset distribution remains symmetric. Moreover, given the unit upper bound on holdings of assets, there is only one symmetric asset distribution at any date consistent with the total amount of the fiat asset being held: if m is the per type amount of it at the start of a date, then a fraction m of each type has a unit of it and a fraction $1 - m$ has nothing. It follows that the sequence of symmetric asset distributions is simple, provided people do not discard the fiat asset: the date 0 distribution is the unique symmetric one with $m = m_0$, and the distribution at all other dates is the unique symmetric one with $m = m_0 + \Delta$. This simple dependence of the fiat asset distribution on the asset's quantity is one of the main simplicities achieved by the assumption that the fiat asset is indivisible and that there is a unit upper bound on individual holdings.

Given symmetric asset distributions, if m_t is the amount of the fiat asset at the start of date t, then the fraction of all meetings that are trade meetings is $(2/N)(1-m_t)m_t$, where $2/N$ is the fraction of all meetings that are single-coincidence meetings and $(1-m_t)m_t$ is the fraction of those in which the producer does not have the fiat asset and the consumer does.

The Equilibrium Effects

I now describe what happens in the trade meetings at each date and how what happens depends on Δ, the quantity of the fiat asset discovered at the end of date 0. The following claim, demonstrated in Wallace 1996b, asserts the existence of and describes the unique equilibrium in which the fiat asset has a positive and constant value starting at date 2.

Claim 1. An equilibrium with the following features exists:

- The fiat asset is valuable at every date.
- The equilibrium is constant starting at date 2, and the constant price level at $t \geq 2$ is increasing in Δ.
- At date 1, the price level does not depend on Δ, and total output is increasing in Δ and varies with Δ more strongly than at date 2 and thereafter.

By *long-run* effects of changes in the quantity of the fiat asset, I mean the dependence on Δ of what happens in that equilibrium at date 2 and thereafter. By *short-run* effects, I mean the dependence on Δ of what happens in the Claim 1 equilibrium at date 1. In other words, long-run effects are those in a date 2 cross section, while short-run effects are those in a date 1 cross section—cross sections from economies that are identical except for the realization of Δ. Date 2 is the long run because the equilibrium is constant starting at that date. Date 1 is the short run because it immediately follows the change in the quantity of the fiat asset.

To explain Claim 1, I start by associating magnitudes in the Claim 1 equilibrium with the price level and total output. As explained further below, for a given realization of Δ and a given date t, the amount produced in each trade meeting is the same, an amount denoted $c_t(\Delta)$. Since $c_t(\Delta)$ exchanges for one unit of the fiat asset in every trade, the price level at date t and for a given realization of Δ is $1/c_t(\Delta)$. In regard to total output, I noted above that the fraction of all meetings which are trade meetings at date t is $(2/N)(1-m_t)m_t$ and that at date 1 and thereafter, $m_t = m_0 + \Delta$. It follows that total output is the product of that fraction, N (the number of types), and $c_t(\Delta)$. Thus, if $Y_t(\Delta)$ denotes total output at date t and for a given realization of Δ, then for $t \geq 1$,

$$(1) \qquad Y_t(\Delta) = 2(1-m_0-\Delta)(m_0+\Delta)c_t(\Delta).$$

Because $m_0 + \Delta \leq 1/2$, $(1-m_0-\Delta)(m_0+\Delta)$ is increasing in Δ. That is, the fraction of all meetings that are trade meetings is increasing in Δ. That is a source of expansionary real effects of changes in the amount of the fiat asset.

The Long Run

At the start of date 2, everyone knows Δ. Thus, starting at date 2, the model has a constant and known amount of the fiat asset per type. That is why there is an equilibrium in which what happens in each trade meeting is constant from date 2 onward. In this equilibrium, the amount produced in each trade meeting at date t for $t \geq 2$ is decreasing in m_t, because at any such date t, the probability of a consumer meeting a producer without the fiat asset is $(1-m_t)/N$. Hence, the greater is m_t, the lower is that probability. Therefore, a producer, looking ahead to that probability, is willing to produce less to acquire the fiat asset the larger is m_t. Therefore, for $t \geq 2$, $c_t(\Delta)$ is decreasing in Δ. Because the price level is the inverse of the amount produced, it follows that for $t \geq 2$, the price level is increasing in Δ. In addition, because the only future gain to a producer is the possibility of consuming $c_t(\Delta)$ and because the producer discounts the future, it must be that the utility of consuming $c_t(\Delta)$, $u(c_t(\Delta))$, exceeds the disutility of producing $c_t(\Delta)$, which implies that $c_t(\Delta) < x^*$. (See the figure.)

By equation (1), total output is the product of two functions. One function is the probability of a trade meeting occurring. That part, given by $2(m_0+\Delta)(1-m_0-\Delta)$, is identical at $t = 1$ and $t \geq 2$ and, under my assumption about the range of Δ, is increasing in Δ. The other function is the amount produced in a trade meeting, $c_t(\Delta)$. At $t \geq 2$, $c_t(\Delta)$ is decreasing in Δ. Therefore, the model makes no definite qualitative prediction about how total output at date 2 and thereafter varies with Δ.

The Short Run

Each producer in a trade meeting at date 1 knows how the value of the fiat asset in a trade meeting at date 2 and thereafter varies with Δ. In other words, each knows what the long run is like for a given realization of Δ. But no producer at date 1 knows what Δ is. Therefore, each acts on the basis of an expected value—the expected value of the expected discounted utility of starting date 2 with a unit of the fiat asset. The expected value is computed using the producer's posterior distribution over Δ. That posterior distribution is arrived at using Bayes' rule, the prior distribution given by p_i, and the experience of each producer.

Each producer in a trade meeting at date 1 has been through the same experience: each left a meeting at date 0 without the fiat asset, did not discover a unit of it, and met someone with a unit of it. The producer does not know whether the consumer left a meeting at date 0 with the fiat asset or left without the fiat asset and subsequently discovered a unit of it.[8] Therefore, the posterior distribution of each producer is the same. Moreover, each producer's trading partner knows the producer's experience and, therefore, the producer's posterior distribution. It follows that the maximum amount produced in every trade meeting at date 1 is the same and does not depend on Δ.[9] Therefore, I denote it c_1. In fact, c_1 is a weighted average of the amounts produced at date 2 for each possible realization of Δ, the $c_2(\Delta)$, with weights given by the common posterior distribution of producers at date 1.

Finally, I must argue that each consumer in a trade meeting at date 1 is willing to give up the fiat asset for c_1. There are two kinds of consumers, those who left a meeting at date 0 with the fiat asset and those who left without the fiat asset and discovered a unit of it. The two kinds of consumers have different posterior distributions over Δ—distributions that differ from those of producers at date 1. To ensure that both kinds of consumers want to trade the fiat asset for c_1, I assume that the range of Δ is small enough so that both kinds of consumers will want to trade at date 1 even if the producer's posterior distribution puts a weight of unity on the largest possible Δ and consumers put a weight of unity on the smallest possible Δ. (One way to see that there is always such a range is to notice that if there is no uncertainty about the aggregate amount of the fiat asset discovered, then what happens at dates 1 and 2 will be the same.)

Because all trade at date 1 consists of the exchange of c_1 for one unit of the asset, the price level at date 1 is $1/c_1$ and, therefore, does not depend on the realization of Δ. A consequence is that total output at date 1 is increasing in Δ, because $c_1(\Delta)$ in equation (1) does not depend on Δ and because the fraction of meetings that are trade meetings is increasing in Δ. Finally, even if total output is increasing in Δ at date 2, it follows from (1) and the conclusions about

$c_2(\Delta)$ and $c_1(\Delta)$ that total output varies more strongly with Δ at $t = 1$ than at $t \geq 2$.[10]

Although I could also describe what happens at date 0 meetings and thereby complete the description of the Claim 1 equilibrium, there is no reason to do that here. The long-run vs. short-run comparisons I am after are given by the descriptions above. In the long run, the price level is increasing in the aggregate discovery of the fiat asset, while total output could be increasing (but less strongly than in the short run), could be decreasing, or could be nonmonotone in the aggregate discovery of it. In the short run, the price level does not depend on the aggregate discovery of the fiat asset, while total output is increasing in the aggregate discovery of it.

Robustness

Although the model contains many extreme assumptions, two deserve special attention: the public knowledge at the start of date 2 about the realized change in the quantity of the fiat asset and the indivisibility of the fiat asset with the upper bound on individual holdings.

The assumption that everyone learns the realized change in the quantity of the fiat asset with a one-date lag is made to keep the model simple. The natural assumption is that people never learn the realized change, but only draw inferences about it based on experience. I see two difficulties with that assumption or even with an assumption that lengthens the lag with which everyone learns the realization. One difficulty is that prior distributions over Δ get revised in accord with experience (at least experience regarding what the trading partner has). Because experience is diverse, an analysis would have to keep track of groups that are diverse in terms of their posterior distributions over the change in the amount of the fiat asset. The other difficulty is that the bargaining would then be between two people who do not know each other's posterior distributions. Despite these difficulties, it is plausible that the qualitative features I find for the one-date information lag formulation would continue to hold, but not in the same way. Under the natural assumption, people would eventually (in the limit) learn the realization, because the probability of meeting someone with the fiat asset is $m_0 + \Delta$ and because the frequency of such meetings becomes a better estimate of that probability as time passes. Therefore, there ought to be an equilibrium that converges to what happens at date 2 under the one-date information lag formulation. Moreover, although the implied short run would then merge smoothly into the long run, rather than end abruptly after one date as under my assumption, the effects would again be entirely real at date 1 and mainly real for several dates thereafter.

The assumption that the fiat asset is indivisible and that there is a unit upper bound on individual holdings is also made to keep the model simple. One implication of that formulation is that those who discover the fiat asset are not producers at date 1: either they are consumers or they do not trade—that being a consequence of the indivisibility and the upper bound. If there is no upper bound, then the discovery process can be random among everyone. Given such randomness, total output at date 1 may not be increasing in the aggregate discovery, because producers who have discovered the fiat asset will tend to produce less.[11] One way to amend the model to restore such dependence is to allow people some choice about whether to produce or to consume. If there is such choice, then those who discover the fiat asset will tend to be consumers. Such a choice appears in some closely related matching models, but those models also include the indivisibility and unit upper bound assumption. (See Diamond 1984 and Kiyotaki and Wright 1991.)

My final comment on the assumptions is to point out similarities between the model and the ingredients in the following passage from Hume (1752, pp. 37, 38):

> Accordingly we find, that, in every kingdom, into which money begins to flow in greater abundance than formerly, every thing takes a new face: labour and industry gain life; the merchant becomes more enterprising . . . .
>
> To account, then, for this phenomenon, we must consider, that though the high price of commodities be a necessary consequence of the encrease of gold and silver, yet it follows not immediately upon that encrease; but some time is required before the money circulates through the whole state, and makes its effect be felt on all ranks of people. At first, no alteration is perceived; by degrees the price rises, first of one commodity, then of another; till the whole at last reaches a just proportion with the new quantity of specie which is in the kingdom. In my opinion, it is only in this interval or intermediate situation, between the acquisition of money and rise of prices, that the encreasing quantity of gold and silver is favourable to industry. When any quantity of money is imported into a nation, it is not at first dispersed into many hands, but is confined to the coffers of a few persons, who immediately seek to employ it to advantage. Here are a set of manufacturers or merchants, we shall suppose, who have received returns of gold and silver for goods which they sent to Cadiz. They are thereby enabled to employ more workmen than formerly, who never dream of demanding higher wages, but are glad of employment from such good paymasters . . . . [The artisan] carries his money to market, where he finds every thing at the same price as formerly, but returns with greater quantity and of better kinds, for the use of his family. The farmer and gardener, finding, that all their commodities are taken off, apply themselves with alacrity to the raising more . . . . It is easy to trace the money in its progress through the whole commonwealth; where we shall find, that it must first quicken the diligence of every individual, before it encrease the price of labour.

The model and Hume's discussion are similar in that both contain two, and only two, main explanatory ingredients: trade is decentralized in the sense that not everyone is together, and information about the quantity of money is incomplete in that, at most, some people, those "who have received returns of gold and silver for goods which they sent to Cadiz," know that it has increased.[12]

The Policy Implications

The implications of the model of the disparate long- and short-run effects of changes in the quantity of money are much like those of other models which rely on uncertainty about the quantity of money and incomplete information about realizations of changes in the quantity. The model implies that the short-run effects depend on those conditions and should not be expected to occur for changes in the quantity of money that do not satisfy those conditions. In particular, if the change in the aggregate amount of money were known by everyone at date 1, then there would be only long-run effects. That is, my description of what happens at date 2 and thereafter would hold for date 1 and thereafter. Obviously, this holds for the case of a degenerate prior distribution of changes in the quantity of money—at date 0, everyone knows what the aggregate change will be.

Hume, by the way, can be accused of failing to recognize the importance of the asymmetric and incomplete information about the quantity of money that is present in his passage above. Hume (1752, pp. 39, 40) concludes his discussion of the effects of increases and decreases in the quantity of money as follows:

> From the whole of this reasoning we may conclude, that it is of no manner of consequence, with regard to the domestic happiness of a state, whether money be in a greater or less quantity. The good policy of the magistrate consists only in keeping it, if possible, still encreasing; because, by that means, he keeps alive a spirit of industry in the nation, and encreases the stock of labour, in which consists all real power and riches.

The model can be used to appraise Hume's policy recommendation if his recommendation can be translated into one concerning m_0 and the distribution of Δ. The choice of a recommendation concerning m_0 and the distribution of Δ is consistent with the model if I suppose, as seems plausible, that a society can choose its money from among fiat objects—objects with different supply conditions in the form of different m_0's and distributions of Δ. (The objects not chosen, if they are fiat objects, can be valueless in equilibrium.) Moreover, if I judge the consequences of different m_0's and distributions of Δ as of date 0 before people know whether they do or do not start out with a unit of the fiat asset, then those consequences can be judged using a representative agent welfare criterion, $(1-m_0)v_0(0) + m_0v_0(1)$, where $v_0(i)$ is the expected discounted utility of starting date 0 with i units of the fiat asset. In Wallace 1996b, I report one numerical example in which I hold m_0 fixed and compare distributions with the same mean and different variances. In that example, representative agent welfare is decreasing in the variance. Hume seems to favor a distribution of Δ with a positive mean to one with no change in the quantity of money ($\Delta \equiv 0$).

The Coexistence Challenge

The coexistence of money and higher-return assets is the other main challenge facing monetary theory. Hicks (1935) called attention to the coexistence about 60 years ago, and today monetary theorists still regard it as a challenge. (See, for example, Hellwig 1993.) A standard result in economics is that rates of return on different assets of equal riskiness tend to get equalized. The coexistence challenge is to explain why money is free from this equalization tendency. The coexistence is also important because different interpretations of it, different explanations for it, give rise to different policy prescriptions. Some interpretations of the coexistence suggest, for example, that policy ought to be directed toward equalizing the returns on money and other assets through some device for paying interest on money.

A starting point of an explanation for the coexistence is to notice that the standard result that returns tend to get equalized has behind it several assumptions, the most important of which is that all assets are traded on competitive markets to which everyone has access. Therefore, one route—in effect, a necessary route—to explaining the coexistence is to depart from that assumption. The pairwise meeting model set out above does depart from it and in an extreme way. Indeed, using that model is, in a sense, bending over backward to get the coexistence, because the model gives considerable scope to the familiar notion that the use of a particular object as a medium of exchange is a coordinating device and cannot be explained in terms of the intrinsic properties of the object, including its rate of return. (See, for example, Tobin 1980.)[13] Such an idea can even be read into Smith's (1776, pp. 22, 23) absence-of-double-coincidence passage, where he says that "every prudent man . . . must naturally have endeavoured to manage his affairs in such a manner, as to have at all times by him . . . a certain quantity of some one commodity or other, such as he *imagined* [emphasis added] few people would be likely to refuse in exchange for the produce of their industry." My model gives scope to that coordination idea, because people in my model have to think about, or imagine, what others will subsequently accept from them. Therefore, my model gives scope to the possibility that a belief that the higher-return asset is not accepted in trade is a self-sustaining belief.

The Dividend-Bearing Asset

I now assume that there are two distinct assets. One asset is identical to the fiat asset described earlier. The other asset is similar, except that it throws off a dividend, denoted ρ, per period. I call this second asset the ρ *asset.* I assume that the dividend, ρ, is in the form of a perishable good that is distinct from all the other N goods and is a perfect substitute for the consumption good of any type. I let A_m denote the constant amount of the fiat asset per type and let A_ρ denote the constant amount of the ρ asset per type. I assume that $A_m + A_\rho < 1$. I also assume that the initial asset distribution is symmetric across types of people so that, initially, A_m is the fraction of each type who begin with a unit of the fiat asset and A_ρ is the fraction of each type who begin with a unit of the ρ asset.

The Equilibrium Concept

Here, as in the model with one asset, an *equilibrium* is a description for each date, starting at date 0 and stretching into the indefinite future, of the frequencies with which different kinds of meetings occur and of what happens in those meetings. The sequence of actions at each date is as follows. At the start of each date, each person has either one unit of one of the assets or nothing. Then those who start with a unit of the ρ asset realize and consume the dividend ρ. (Thus, I am not allowing dividends, as distinct from assets, to be traded.) Next, people meet in pairs at random and bargain. Because of the upper bound of unity on individual holdings of assets and because of the assumed symmetry across types of people, here, as in the model with one asset, there is a potential for trade only in single-coincidence meetings. Now, however, there are two situations in which trade can possibly occur. In one, the producer has no asset and the consumer has an asset—either the fiat asset or the ρ asset. In the other, the consumer has a more valuable asset than the producer has and, therefore, can offer the more valuable asset and acquire in exchange some production and the less valuable asset. If the outcome of bargaining implies exchange, then production and consumption occur. In regard to bargaining, I continue to assume that the potential consumer makes a take-it-or-leave-it offer and that the potential producer accepts if made no worse off by accepting. Thus, as above, this take-it-or-leave-it bargaining and rational expectations constitute the equilibrium concept.

Again, such take-it-or-leave-it bargaining and rational expectations do not completely pin down what happens. I restrict attention to equilibria that are symmetric across types of people. In this model, as in the model with one asset, if the initial asset distribution is symmetric and trades are symmetric, then the asset distribution remains symmetric. Also, as above, here there is only one symmetric asset distribution consistent with all assets being held, namely, the initial symmetric asset distribution. That being so, the frequency with which various kinds of meetings occur is determined by the asset amounts and N.

The Equilibrium Effects

As was true in the model with one asset, there is an equilibrium here in which the fiat asset has no value. In what follows, I ignore this equilibrium. In regard to equilibria in which the fiat asset has value, any such constant equilibrium will display the coexistence I am after. In such a constant equilibrium, the yield on the fiat asset, either real or nominal, is zero. If the ρ asset is traded, then its yield is ρ divided by the amount of production for which it trades and is, therefore, positive. If the ρ asset is not traded, then it has a *bid price*, what someone is willing to produce in order to acquire it, and an *ask price*, what someone will demand in order to give it up. When either of those prices is used, the yield is, again, positive. However, I am interested in more than such coexistence. I show that the magnitude of the yield on the ρ asset is associated with the frequency with which it is traded: a lower yield is associated with a higher trading frequency.

Although all the parameters in the model determine the kind of equilibria that can arise, I describe how the equilibria depend on ρ when all the other parameters are held constant. If ρ is sufficiently close to zero, then the ρ asset functions as another valued fiat asset. That is, it is traded frequently, and its yield approaches zero as ρ approaches zero. If, instead, ρ is sufficiently large, then it is not traded, and its yield is higher than when ρ is close to zero. Moreover, in the first case, the yield on the ρ asset is affected by the quantity of the fiat asset, while in the second case, it is not. I describe these results in more detail as a list of claims.[14]

I begin with a preliminary discussion of a necessary condition for the ρ asset not to be traded. A holder of a unit of the ρ asset always has the option of holding it forever and never trading it. In that case, the holder of the ρ asset consumes ρ at every date and thereby realizes utility equal to $u(\rho)$ at every date. The result is an expected discounted utility of $u(\rho)/(1-\beta)$. In a constant equilibrium with the take-it-or-leave-it bargaining rule, the expected discounted utility of having no asset is zero. Therefore, if a holder of the ρ asset meets a potential producer with no asset, then that producer is willing to produce at least $\beta u(\rho)/(1-\beta)$ to acquire the ρ asset, because that producer can, at worst, hold the ρ asset forever. Therefore, a necessary condition for a holder of the ρ asset to refuse to trade is

$$(2) \qquad u(\rho)/(1-\beta) \geq u\big(\rho + [\beta u(\rho)/(1-\beta)]\big)$$

where the left side is what the holder gets by never trading the asset and the right side is the minimum of what the holder gets by offering the ρ asset to a producer with no asset. It is easy to demonstrate that this condition fails for ρ sufficiently close to zero and holds for ρ sufficiently large. With this as background, I describe, in turn, the constant equilibria when ρ is sufficiently close to zero and when ρ is sufficiently large. In what follows, I let c_m denote the amount produced in exchange for a unit of the fiat asset and let c_ρ denote the amount produced in exchange for a unit of the ρ asset. I also let $c_{\rho m}$ denote the amount produced when the producer acquires the ρ asset and surrenders the fiat asset and let $c_{m\rho}$ denote the amount produced when the producer acquires the fiat asset and surrenders the ρ asset.

□ *Small Dividends*

I arrive at the results in this case by starting with $\rho = 0$ and using continuity to draw conclusions about what happens with ρ close to 0. Therefore, I begin by describing the symmetric and constant equilibria with both assets valuable when $\rho = 0$.

I assume that the two assets are distinguishable by something irrelevant like their colors, even if $\rho = 0$. But if $\rho = 0$, then there is an equilibrium in which the two assets are treated as indistinguishable.

Claim 2. If $\rho = 0$, then there is exactly one symmetric and constant equilibrium with $c_m = c_\rho > 0$. (I let c^* denote this common positive value of c_m and c_ρ.) In this equilibrium, $c_{\rho m} = c_{m\rho} = 0$.

This equilibrium is identical to the one in the long-run part of the Claim 1 equilibrium if $m_0 + \Delta = A_m + A_\rho$.

Next, I let ρ be positive, but close to zero. There is an equilibrium in which the value of each asset is close to c^*.

Claim 3. If ρ is positive and sufficiently close to zero, then there is an equilibrium near the Claim 2 equilibrium with $c_m(\rho) > c_\rho(\rho) > c^*$ and $c_{m\rho}(\rho) > 0$. (This equilibrium is near the Claim 2 equilibrium in the sense that $c_m(\rho) \to c^*$ and $c_{m\rho}(\rho) \to 0$ as $\rho \to 0$.)

Notice three things about this equilibrium. First, the fiat asset is more valuable than the ρ asset. (There is no constant equilibrium near the Claim 2 equilibrium in which the ρ asset is at least as valuable as the fiat asset.) Second, trade occurs in all the potential trading situations described above. In particular, trade occurs in single-coincidence meetings when the producer has no asset and the consumer has either asset and when the producer has the ρ asset and the consumer has the fiat asset. Thus, the ρ asset functions as a second valuable fiat asset. Third, the yield on the ρ asset approaches zero as ρ approaches zero. Moreover, the yield on the ρ asset depends on the quantity of the fiat asset, A_m, because the yield on the ρ asset is approximately equal to ρ/c^* and because c^* depends on A_m.

When $\rho = 0$, the Claim 2 equilibrium is not the only equilibrium in which both assets have value. There are two other equilibria.

Claim 4. If $\rho = 0$, then there is a symmetric and constant equilibrium with $c_\rho = c^*$ and $c_{m\rho} > 0$, and there is a symmetric and constant equilibrium with $c_m = c^*$ and $c_{\rho m} > 0$, where c^* is defined in Claim 2.

These Claim 4 equilibria can be described as *endogenous denomination equilibria.*[15] In each of them, one of the assets is treated as more valuable than the other. There are two such equilibria, because either asset can be treated as the more valuable asset. (If $A_m = A_\rho$, then the two Claim 4 equilibria are identical, except for the irrelevant labeling of which is more valuable. If $A_m \neq A_\rho$, then the two equilibria are not identical.) Notice that the less valuable asset in these equilibria has the value that both assets have in the Claim 2 equilibrium.

Trade occurs if the producer has no asset and the consumer has the less valuable asset and if the producer has the less valuable asset and the consumer has the more valuable asset. Trade may or may not occur if the producer has no asset and the consumer has the more valuable asset. If the discount factor is sufficiently close to one, then trade does not occur in those situations. In effect, the consumer is not willing to spend all of his or her wealth at once. If the discount factor is sufficiently close to zero, then trade does occur.

The next claim describes analogs of the Claim 4 equilibria when ρ is positive and close to zero.

Claim 5. If ρ is positive and sufficiently close to zero, then there are equilibria that are close to the Claim 4 equilibria. In particular, there is a constant equilibrium with $c_\rho(\rho) > c^*$ and $c_{m\rho}(\rho) > 0$, and there is a constant equilibrium with $c_m(\rho) = c^*$ and $c_{\rho m}(\rho) > 0$. (These equilibria are near the Claim 4 equilibria in the sense that $c_\rho(\rho) \to c^*$, $c_{m\rho}(\rho) \to c_{m\rho}$, and $c_{\rho m}(\rho) \to c_{\rho m}$ as $\rho \to 0$, where $c_{m\rho}$ and $c_{\rho m}$ are given in Claim 4.)

Claim 5 can be summarized by saying that a small positive dividend can be attached either to the less valuable asset in Claim 4 or to the more valuable asset without much affecting the equilibrium. If the dividend is attached to the less valuable asset, then its yield is approximately ρ/c^*. If it is attached to the more valuable asset, then its yield is approximately $\rho/(c^*+c_{\rho m})$. In either case, the yield approaches zero as the dividend approaches zero. Also, here, as was true for the equilibrium of Claim 3, the yield depends on the quantity of the fiat asset.

Large Dividends

As suggested above, if ρ is sufficiently large, then holders of the ρ asset do not trade. In particular, if ρ satisfies a strengthened version of inequality (2), then there is a unique constant equilibrium in which the fiat asset has value, and it is one in which holders of the ρ asset do not trade.

Claim 6. Let x^* be the unique positive solution to $u(x^*) = x^*$. (See the figure.) If ρ is such that

$$(3) \qquad u(\rho)/(1-\beta) \geq u\big(\rho + [\beta u(\rho)/(1-\beta)]\big) + x^*$$

then there is a unique symmetric and constant equilibrium in which the fiat asset has value. It is one in which holders of the ρ asset do not trade and in which $c_m(\rho) = c^*$.

In the Appendix, I show that all sufficiently large ρ satisfy inequality (3). The existence part of Claim 6 is established by construction. If only the fiat asset is traded, then c_m does not depend on ρ and is equal to c^* in Claim 2 where, of course, $c^* < x^*$. Given the implied expected discounted utility of starting a date with the fiat asset, which is βc^*, and given the expected discounted utility of starting a date with the ρ asset when it is not traded, $u(\rho)/(1-\beta)$, the existence claim follows if a potential consumer with the ρ asset will not want to give it up to a potential producer who either has no asset or has the fiat asset. As shown in the Appendix, inequality (3) is sufficient for that to be true. The uniqueness claim follows from the fact that the left side of inequality (3) is a lower bound on the expected discounted utility of starting a date with a unit of the ρ asset.

If the ρ asset is not traded, then it does not have a price in the sense of the amount of the good exchanged for it. It does, however, have a bid price, $\beta u(\rho)/(1-\beta)$, and an ask price, the solution for x to $u(\rho+x) = u(\rho)/(1-\beta)$. I can, therefore, measure the yield on the ρ asset as the ratio of ρ to one of those prices. When either price is used, that yield does not depend on A_m, the amount of the fiat asset, and is higher than the yield on the ρ asset when ρ is close to zero.[16]

Yields and Trade

Because the above claims deal with only very small and very large dividends, they do not constitute a complete description of even the symmetric and constant equilibria. Nevertheless, the claims are sufficient for my purposes. They show that the role of the ρ asset in trade and its yield depend on its physical characteristics. If the dividend is sufficiently close to zero, then the ρ asset functions like a second valuable fiat asset. If, instead, the dividend is very large, then the ρ asset does not. In regard to its yield, the real yield is lower if the dividend is sufficiently small than if it is sufficiently large. Thus, as asserted above, the yield on the ρ asset is associated with the frequency with which the asset is traded: a lower yield is associated with a higher trading frequency.

Finally, it follows that anticipated inflation and the yield on the ρ asset are associated in different ways in the two cases. The specification in the model with only the fiat asset permits me to analyze the effects of a one-time nonrandom change in the amount of the fiat asset that produces an anticipated inflation. If ρ is close to zero, then the real yield on the ρ asset depends on that anticipated change in the amount of the fiat asset. If, instead, ρ is sufficiently large, then the real yield on the ρ asset does not depend on the change. All of this holds for changes in the amount of the fiat asset that are small enough to be consistent with the fiat asset being valuable.

In a sense, these results are unsurprising. After all, if the dividend is sufficiently small, then the ρ asset is physically like the fiat asset; otherwise, the ρ asset is not. Therefore, these results show only that if the ρ asset is physically sufficiently like the fiat asset, then there are equilibria in which the ρ asset functions like the fiat asset; otherwise, it does not. Such results are surprising only against the background of most of existing monetary theory. As emphasized in Wallace 1996a, most of existing monetary theory has no implications for the relationship between the physical characteristics of objects and their role in trade.

Robustness

I have shown that the fiat asset can have value despite the existence of the ρ asset. That result may be due, however, to the assumed asset indivisibility with a unit upper bound on individual holdings of assets. That assumption permits only very indirect competition between the fiat asset and the ρ asset. For example, no one is ever in the position of having both assets and choosing which to offer. In that sense, the mere finding of coexistence is not surprising.

Not much is known about equilibria under more general assumptions about individual asset holdings. My guess is that indivisibility of the ρ asset is crucial for the coexistence, but that the bound on individual holdings is not. That is, my guess is that if the ρ asset were perfectly divisible, then its existence in any positive amount would be inconsistent with a positive value for the fiat asset. If that were so, then I would have another instance of the result that the physical characteristics of the assets have implications for their values and for their roles in trade. Again, that seems surprising only against the background of most of existing monetary theory.

I do not mean to suggest by this discussion that perfect divisibility is the only assumption of interest. Divisibility has long been regarded as a desirable property of a medium of exchange. Divisibility would not warrant mention if it were not rare, that is, if most objects were perfectly divisible.

The Policy Implications

The standard view among economists is that coexistence of money and higher-return assets is a symptom of nonoptimality. (See, for example, Samuelson 1968, Friedman 1969, and Lucas 1986.)[17] Friedman (1969) is generally credited with suggesting a remedy for this nonoptimality, one which has come to be known as the *Friedman rule:* Pay interest on money either explicitly or by generating enough deflation to make the nominal interest rate zero; in either case, finance the interest through taxation. (See Friedman 1960, 1969.) The model above does not have that implication if only because, as the model stands, taxation is not feasible. That, however, should not be regarded as a defect of the model.

A discussion of policy should include defensible claims about the policies that are feasible. And no one should be surprised if the restrictions needed to give monetary trade a role limit the range of feasible policies. Conversely, assumptions about feasible policies may be inconsistent with the restrictions needed to give monetary trade a role.

For example, suppose I amended the model in the following extreme way so that it is consistent with taxation. Suppose there is an additional person, a disinterested public servant, who participates in each pairwise meeting between the other people and who can tax them. Instead of taxing to pay interest on money, the public servant could simply direct production and consumption in each single-coincidence meeting as could happen if people could commit themselves to future actions. In particular, in each single-coincidence meeting in which the consumer has consumed ρ because the consumer starts with the ρ asset, the producer could be directed to produce and give $\max(0,y^*-\rho)$ to the consumer. In all other single-coincidence meetings, the producer could be directed to produce and give y^* to the consumer, where y^* satisfies $u'(y^*) = 1$. (See the figure.) That good outcome, as judged by the welfare of people at date 0, could then be achieved without using money.

Less Extreme Assumptions

The results above should make economists less skeptical about the value of models built on the absence-of-double-coincidence idea. Those results demonstrate that such models are well-suited to confront two long-standing challenges in monetary economics: the disparate long- and short-run effects of changes in the quantity of money and the coexistence of money and higher-return assets. However, those results are obtained in a model which, while built on the absence-of-double-coincidence idea, seems very extreme. In order for such models to win wide acceptance, they must be amenable to generalizations that make them less extreme. Happily, they are. Here I describe particular ways of generalizing three of the extreme assumptions in the model: the indivisibility of assets with a unit upper bound on individual holdings, pairwise meetings at random, and private information about the history of each person's actions except insofar as that history is conveyed by the person's current holdings of assets.

In regard to individual asset holdings, an alternative extreme is perfectly divisible assets with no bound on individual holdings. A one-dimensional way to fill the gap between that extreme and the specification in the model is to vary the degree to which assets are indivisible and to vary the upper bound on asset holdings. In particular, suppose B is a positive integer that is the upper bound on individual holdings of indivisible assets. Suppose also that the amount of each indivisible asset is proportional to B and that the dividend of each asset is inversely proportional to B. Then total dividends are independent of B, $B = 1$ is the specification adopted above, and divisibility with no upper bound is approached as $B \to \infty$. As I noted above, my conjecture is that if there is a positive dividend-bearing asset, then the use of a fiat object does not survive in the limit as $B \to \infty$. Of course, divisibility is only one among many physical characteristics that could differ among assets and make some more suitable in trade than others.

In regard to pairwise meetings at random, an alternative extreme is that everyone is together at every date. If competitive equilibrium is taken to be the equilibrium concept with everyone together, then the unique equilibrium with everyone together has relative prices of all goods at each date equal to unity, has a constant real interest rate equal to $\beta^{-1} - 1$, has a zero value of the fiat asset, has each person without a dividend-bearing asset consuming and producing y^*, and has each person with a dividend-bearing asset producing $\max(0,y^*-\rho)$ and consuming $\max(y^*,\rho)$. (See the figure.) A simple one-dimensional way to fill the gap between the extreme with everyone together and pairwise meetings at random is to have meetings of J people at random. In general, that permits some double coincidences. If the only asset is a fiat asset, then a plausible conjecture is that there is some role for the fiat asset for any finite J, but that its value tends to zero as $J \to \infty$.

In regard to privacy of the history of people's actions, the alternative extreme is public knowledge of every person's history. As demonstrated in Kocherlakota 1996, if there is public knowledge of every person's history, then the use of outside assets for trade is inessential in the sense that any allocation achievable using outside assets for trade is also achievable without using them. A one-dimensional way of filling the gap between the two extremes of no public knowledge and complete public knowledge is to assume that at each date t, every person's history up to $t - T$ is public knowledge for some nonnegative integer T. Then $T = \infty$ is no public knowledge, while $T = 0$ is complete public knowledge. I surmise that there would be some role for both outside assets in trade and some role for some form of credit for at least some positive and finite magnitudes of T. Kocherlakota and Wallace 1996 demonstrates that this is true for a closely related formulation.[18]

Concluding Remarks

Although deriving conclusions from models generalized as just indicated or in other ways may be difficult, the above specifications show that there is nothing inherently extreme about models built on the absence-of-double-coincidence idea. Therefore, it seems that a suitable remedy for the schizophrenia that has for so long afflicted monetary economics is now available. Economists are now able to formulate a general class of models consistent with the long-held belief that the use of a medium of exchange is the result of real frictions that give rise to absence-of-double-coincidence problems. As I have demonstrated, a particular model in this class can account for the two main challenges facing monetary theory: the disparate long-run and short-run effects of changes in the quantity of money and the coexistence of money and assets with higher rates of return. One form that further progress will take is the detailed study of the general class of models.

[1]The quantity theory equation, in symbols, is $MV = PY$, where M is the assumed exogenous quantity of money, V is the assumed exogenous velocity of money, P is the endogenous price level, and Y is real income, which is determined by the general competitive equilibrium part of the model.

[2]In a footnote, Samuelson (1968, p. 179) says, "This [money as an argument of utility functions] is not the only way of introducing the real convenience of cash balances. An even better way would be to let U [utility] depend only on the time stream of qs [quantities of ordinary goods], and then to show that holding an inventory of M [money] does contribute to a more stable and greatly preferable stream of consumptions."

[3]Otherwise, why can't Mishkin's (1986) Ellen get food by issuing IOUs that promise delivery of brilliant economics lectures—IOUs which themselves get traded until they end up in the hands of those who want brilliant economics lectures and present them as payment to Ellen (it being understood that at least someone values brilliant economics lectures)?

[4]The assumption that the disutility of production is equal to the amount produced is without loss of generality. For details, see Aiyagari, Wallace, and Wright 1996.

[5]The role of this assumption and the inability to commit is emphasized in Aiyagari and Wallace 1991. See also Huggett and Krasa 1996 and Kocherlakota 1996.

[6]A version in which everyone can discover a unit of the fiat asset would differ only in insignificant details. Alternatively, a version in which, after trade at date 0, people choose whether to expend some small amount of effort in order to be eligible to discover a unit would not differ at all.

[7]One way to eliminate that equilibrium, while preserving the equilibrium in which the fiat asset has value, is to assume that the fiat asset yields a small amount of utility when consumed. That kind of assumption is used in Sargent and Wallace 1983.

[8]A version of my model in which producers can distinguish between the two kinds of consumers because newly discovered money looks different from old money for one date gives rise to similar qualitative predictions for long- and short-run effects, but has different amounts produced in meetings with the two different kinds of consumers. For details, see Wallace 1996b.

[9]Consumers who did not discover a unit of the fiat asset want to signal that fact to producers. By limiting their strategies to the naming of an amount of production to demand, I do not permit them to do that.

[10]If the support of Δ is specified to be an interval, then the derivative of $Y_2(\Delta)$ with respect to Δ evaluated at the magnitude of Δ at which $c_2(\Delta) = c_1$ is less than the derivative of $Y_1(\Delta)$.

[11]The assumption that new money goes to consumers appears in many other models. See, for example, Lucas 1972, Lucas and Woodford 1993, and Eden 1994. Barro and King 1984 emphasizes the important role of the assumption and questions the rationale for it.

[12]Somewhat surprisingly, in light of the passage cited above, Hume's general discussion of money does not allude to the absence of double coincidence of wants and is limited to the following remark: "[Money] is none of the wheels of trade: It is the oil which renders the motion of the wheels more smooth and easy" (Hume 1752, p. 33).

[13]Indeed, versions of such models are known to be consistent with the use as a medium of exchange of objects which have intrinsic properties worse than those of some other objects. See Kiyotaki and Wright 1989, Aiyagari and Wallace 1992, and Renero 1994, 1995.

[14]These results are related to results in Aiyagari, Wallace, and Wright 1996; however, because they are not identical, proofs of them are given in the Appendix.

[15]The Claim 4 equilibria and the Claim 2 equilibrium can also be interpreted as multiple exchange rate equilibria. It should be evident that if $\rho = 0$ and if the two assets are perfectly divisible, then there is an equilibrium for any constant relative value between them. When the assets are indivisible, the multiplicity is limited, but it still exists.

[16]Evidently, I could have studied a version of my model with three assets: a fiat asset, a ρ asset with ρ close to zero, and a ρ asset with ρ satisfying inequality (3). The result would be a combination of the Claim 6 equilibrium and either a Claim 3 or a Claim 5 equilibrium.

[17]That is not true, however, in all models. See, for example, Woodford 1990 and Levine 1991.

[18]Kocherlakota and Wallace (1996) assume that at each date, all of history becomes public knowledge with some probability, say, γ. The result is an expected number of most recent dates equal to $1/\gamma$ during which what happened is not public knowledge, so that $\gamma = 1$ corresponds to complete public knowledge and $\gamma = 0$ corresponds to no public knowledge. Diamond (1990) and Shi (1995) have versions of random matching models with outside assets and credit, but their formulations are far from straightforward. Diamond permits people who reach a credit agreement to stay together, an option otherwise not available.

Appendix
Proofs of Claims 2–6

Here I develop the proofs for Claims 2–6 presented in the preceding text.

I start with a general setup in which there are two assets, asset 1 and asset 2, each with a dividend. Therefore, I now let ρ_i be the dividend per unit of asset i and let A_i be the amount of asset i per type for $i = 1, 2$. I also let v_i be the expected discounted utility of starting a date with a unit of asset i. (Again, because of the assumed bargaining rule, the expected discounted utility of starting a date with no asset is zero.) Because I am not committed to any special assumption about the dividends, I can, without loss of generality, assume that asset 2 is at least as valuable as asset 1. That is, I can assume that $v_2 \geq v_1$.

The definition below embeds the consequence of the bargaining rule that the consumer demands sufficient production from the producer to keep the producer indifferent between trading and not trading.

DEFINITION. *A symmetric and constant equilibrium is a pair* (v_1,v_2) *that satisfies* $v_2 \geq v_1$ *and*

$$\text{(A1)} \quad v_1 = \alpha\max[u(\rho_1+\beta v_1), u(\rho_1) + \beta v_1] + (1-\alpha)[u(\rho_1) + \beta v_1]$$

$$\begin{aligned}\text{(A2)} \quad v_2 = {} & \alpha\max[u(\rho_2+\beta v_2), u(\rho_2) + \beta v_2] \\ & + (A_1/N)\max\{u[\rho_2 + \beta(v_2-v_1)] + \beta v_1, u(\rho_2) + \beta v_2\} \\ & + [(1-\alpha-A_1)/N][u(\rho_2) + \beta v_2]\end{aligned}$$

where $\alpha = (1-A_1-A_2)/N$.

In equation (A1), α is the probability of any person meeting a potential producer with no asset. Such a meeting gives a holder of asset 1 the option between, on the one hand, realizing a current date utility $u(\rho_1+\beta v_1)$ and starting the next date with no asset (which, as noted above, has an expected discounted utility of zero) and, on the other hand, choosing not to trade. If the first option is chosen, the producer produces βv_1, because that is the producer's discounted gain from acquiring a unit of asset 1. With the remaining probability, a holder of asset 1 gets the payoff from not trading. Although a holder of asset 1 can also trade if he or she meets a holder of asset 2 who consumes what the holder of asset 1 produces, such a trade leaves the holder of asset 1 with the same payoff as not trading. Equation (A2) describes the probabilities and respective options for a holder of asset 2. The second term in that equation represents the options for a holder of asset 2 who meets a potential producer who holds asset 1.

Claims 2–5

I establish these claims by studying the solutions to (A1) and (A2) for ρ_1 and ρ_2 close to zero. Because v_2 does not appear in equation (A1), (A1) can be solved for v_1. Then that solution can be substituted into (A2), which is then solved for v_2.

Solutions to (A1) *for Small* ρ_1

When I let $c_1 \equiv \beta v_1$, and I add and subtract $\alpha[u(\rho_1) + \beta v_1]$ on the right side of (A1), I find that (A1) is equivalent to

$$\text{(A3)} \quad c_1(1-\beta)/\beta = \alpha\max\{u(\rho_1+c_1) - [u(\rho_1) + c_1], 0\} + u(\rho_1) \equiv F(\rho_1,c_1).$$

The function $F(0,c_1)$ is as shown in the accompanying figure. In particular, that function is continuous for $c_1 \geq 0$, is positive and strictly concave for $0 < c_1 < x^*$, and is zero for $c_1 \geq x^*$. Here, as in the preceding text, x^* is the unique positive solution to $u(x^*) = x^*$. (See the figure in the text.) Also, $\partial F(0,0)/\partial c_1 = \infty$. It follows that the equation $c_1(1-\beta)/\beta = F(0,c_1)$ has two nonnegative solutions: $c_1 = 0$ and, by the intermediate value theorem, a second solution that is positive and denoted c^*. (See the figure in this Appendix.)

The function $F(\rho_1,c_1)$ is continuous and increasing in ρ_1. It follows that for $\rho_1 > 0$ and close to zero, there exists a unique solution to equation (A3)—a solution, denoted $c_1(\rho_1)$, which is increasing in ρ_1 and which satisfies $x^* > c_1(\rho_1) > c^*$ and $c_1(\rho_1) \to c^*$ as $\rho_1 \to 0$.

Solutions to (A2) *for Small* ρ_1 *and* ρ_2

When I let $c_{21} \equiv \beta(v_2-v_1)$, and I add and subtract $[(\alpha+A_1)/N] \times [u(\rho_2) + \beta v_2]$ on the right side of (A2), I find that (A2) is equivalent to

$$\begin{aligned}\text{(A4)} \quad & (c_{21}+c_1)(1-\beta)/\beta \\ &= \alpha\max\{u(\rho_2+c_{21}+c_1) - [u(\rho_2) + c_{21} + c_1], 0\} \\ &\quad + (A_1/N)\max\{u(\rho_2+c_{21}) - [u(\rho_2) + c_{21}], 0\} \\ &\quad + u(\rho_2) \\ &\equiv G(\rho_2,c_{21},c_1)\end{aligned}$$

where c_1 should be interpreted as $c_1(\rho_1)$, the positive solution to (A3) described above. The function $G(\rho_2,c_{21},c_1)$ is continuous in c_{21} for $c_{21} \geq 0$. Also, $G(0,c_{21},c_1)$ is positive for $0 \leq c_{21} < x^*$ and is zero for $c_{21} \geq x^*$, where, as above, x^* is the unique positive solution to $u(x^*) = x^*$. Moreover, $\partial G(0,0,c_1)/\partial c_{21} = \infty$, and $G(0,0,c^*) = F(0,c^*)$. It follows that the equation $(c_{21}+c^*) \times (1-\beta)/\beta = G(0,c_{21},c^*)$ has at least two nonnegative solutions for c_{21}: $c_{21} = 0$ and, by the intermediate value theorem, at least one positive solution. The zero solution establishes Claim 2. The positive solution establishes Claim 4 (which is Lemma 2 in Aiyagari, Wallace, and Wright 1996).

Next, notice that $G(\rho_2,c_{21},c_1)$ is continuous and increasing in ρ_2. That and the above results for the positive solution to (A3) establish Claims 3 and 5, as I now explain. I start with Claim 3. Suppose that $\rho_1 > 0$ and $\rho_2 = 0$. Then, by (A3), $F(\rho_1,c_1(\rho_1)) > G(0,0,c_1(\rho_1))$. Therefore, $[0 + c_1(\rho_1)](1-\beta)/\beta > G(0,0,c_1(\rho_1))$. However, since $\partial G(0,0,c_1(\rho_1))/\partial c_{21} \to \infty$ as $\rho_1 \to 0$, there is a solution to equation (A4) with c_{21} positive and close to zero. That is the Claim 3 solution. Notice, however, that if $\rho_1 = 0$ and $\rho_2 > 0$, then there does not exist such a solution, because $(0+c^*) \times (1-\beta)/\beta < G(\rho_2,0,c^*)$.

Now for Claim 5. I noted above that the equation $(c_{21}+c^*) \times (1-\beta)/\beta = G(0,c_{21},c^*)$ has at least one positive solution for c_{21} and that any such solution is less than x^*. Note that $G(0,c_{21},c^*)$ is twice differentiable in c_{21}, except at $c_{21} = x^* - c^*$. Consider the smallest positive solution to $(c_{21}+c^*)(1-\beta)/\beta = G(0,c_{21},c^*)$. This smallest solution is equal to $x^* - c^*$ only for a set of measure 0 in the parameter space and, hence, can be ignored. (Fix all the parameters, including $A = A_1 + A_2$, except A_1. Then c^*, the function $(c_{21}+c^*)(1-\beta)/\beta$, and x^* do not depend on A_1, while G is increasing in A_1. It follows that $c_{21} = x^* - c^*$ is a solution to $(c_{21}+c^*)(1-\beta)/\beta = G(0,c_{21},c^*)$ for only one value of A_1.) Because G has a negative second derivative, except at $c_{21} = x^* - c^*$, it follows that the smallest positive solution to $(c_{21}+c^*)(1-\beta)/\beta = G(0,c_{21},c^*)$ is such that $\partial G(0,c_{21},c^*)/\partial c_{21} < (1-\beta)/\beta$. That and the fact that $\partial F(0,c^*)/\partial c_1 < (1-\beta)/\beta$ allow me to invoke the implicit function theorem, which says that for (ρ_1,ρ_2) in a neighborhood of (0,0), there exists a solution for c_1 and c_{21} that is close to the Claim 3 solution. Claim 5 is a consequence.

Claim 6

This proof has three parts. I first show that inequality (3) in the preceding text holds for all sufficiently large magnitudes of ρ. Then I establish the existence claim and, finally, the uniqueness claim.

Inequality (3) *for Large* ρ

For any $x > 0$ and $y > 0$, $u'' < 0$ implies that $u(x+y) < u(x) + u'(x)y$. Therefore, for any $\rho > 0$,

$$\begin{aligned}\text{(A5)} \quad & u(\rho + [\beta u(\rho)/(1-\beta)]) + x^* \\ &\leq u(\rho) + [u'(\rho)\beta u(\rho)/(1-\beta)] + x^* \\ &= \{u(\rho)[(1-\beta) + u'(\rho)\beta]/(1-\beta)\} + x^* \\ &\equiv h(\rho).\end{aligned}$$

Therefore,

$$\text{(A6)} \quad u(\rho)/(1-\beta) - h(\rho) = \{u(\rho)[1 - u'(\rho)]\beta/(1-\beta)\} - x^*.$$

As $\rho \to \infty$, $u(\rho) \to \infty$ and $u'(\rho) \to 0$. It follows that the left side of equation (A6) is positive for all sufficiently large magnitudes of ρ. Then inequality (A5) implies that inequality (3) holds for all such magnitudes of ρ.

Existence of Equilibrium

In terms of the definition above, the proposed equilibrium is $v_1 = v_m = \beta c^*$ and $v_2 = v_\rho = u(\rho)/(1-\beta)$. Because $c^* < x^*$ and inequality (3) holds, the proposal satisfies $v_2 \geq v_1$. And because Claim 2 implies that the proposal satisfies equation (A1), it remains only to verify that equation (A2) is satisfied. That is, I have to show that

$$\text{(A7)} \quad \max\{u(\rho + [\beta u(\rho)/(1-\beta)]), u(\rho)/(1-\beta)\} = u(\rho)/(1-\beta)$$

$$\text{(A8)} \quad \max\{u(\rho + [\beta u(\rho)/(1-\beta)] - \beta v_m) + \beta v_m, u(\rho)/(1-\beta)\} = u(\rho)/(1-\beta).$$

Because $\beta v_m = c^* < x^*$, inequality (3) implies satisfaction of equations (A7) and (A8).

Uniqueness

If there is another such equilibrium, then it has $v_\rho \geq u(\rho)/(1-\beta)$ and has the ρ asset being traded. The ρ asset must be either asset 1 or asset 2 in the definition. To be asset 1 and be traded, the ρ asset must be offered when the producer has no asset. Therefore, v_ρ must satisfy $u(\rho) + \beta v_\rho \leq u(\rho+\beta v_\rho)$. However, because inequality (3) implies that this inequality fails for $v_\rho = u(\rho)/(1-\beta)$, it fails for all $v_\rho \geq u(\rho)/(1-\beta)$. To be asset 2 in the definition and be traded, the ρ asset must either be traded when the producer has no asset or be traded when the producer has the fiat asset. The former has just been ruled out. The latter implies that v_ρ and v_m satisfy

$$\begin{aligned}\text{(A9)} \quad u(\rho) + \beta v_\rho &\leq u(\rho + \beta(v_\rho-v_m)) + \beta v_m \\ &< u(\rho+\beta v_\rho) + x^*.\end{aligned}$$

Inequality (3) says that inequality (A9) is not satisfied for $v_\rho = u(\rho)/(1-\beta)$. It follows that it is not satisfied for any $v_\rho \geq u(\rho) \div (1-\beta)$.

References

Aiyagari, S. Rao, and Wallace, Neil. 1991. Existence of steady states with positive consumption in the Kiyotaki-Wright model. *Review of Economic Studies* 58 (October): 901–16.

___________. 1992. Fiat money in the Kiyotaki-Wright model. *Economic Theory* 2 (October): 447–64.

Aiyagari, S. Rao; Wallace, Neil; and Wright, Randall. 1996. Coexistence of money and interest-bearing securities. *Journal of Monetary Economics* 37 (June): 397–419.

Barro, Robert J., and King, Robert G. 1984. Time-separable preferences and intertemporal-substitution models of business cycles. *Quarterly Journal of Economics* 99 (November): 817–39.

Diamond, Peter A. 1984. Money in search equilibrium. *Econometrica* 52 (January): 1–20.

___________. 1990. Pairwise credit in search equilibrium. *Quarterly Journal of Economics* 105 (May): 285–319.

Eden, Benjamin. 1994. The adjustment of prices to monetary shocks when trade is uncertain and sequential. *Journal of Political Economy* 102 (June): 493–509.

Friedman, Milton. 1960. *A program for monetary stability.* New York: Fordham University Press.

___________. 1969. The optimum quantity of money. In *The optimum quantity of money and other essays,* pp. 1–50. Chicago: Aldine.

Hellwig, Martin F. 1993. The challenge of monetary theory. *European Economic Review* 37 (April): 215–42.

Hicks, John R. 1935. A suggestion for simplifying the theory of money. *Economica,* N.S. 2 (February): 1–19.

Huggett, Mark, and Krasa, Stefan. 1996. Money and storage in a differential information economy. *Economic Theory* 8 (August): 191–210.

Hume, David. 1752. On money. Reprinted 1970. In *Writings on economics,* ed. Eugene Rotwein, pp. 33–46. Madison: University of Wisconsin Press.

Keynes, John Maynard. 1936. *The general theory of employment, interest, and money.* New York: Harcourt Brace.

Kiyotaki, Nobuhiro, and Wright, Randall. 1989. On money as a medium of exchange. *Journal of Political Economy* 97 (August): 927–54.

___________. 1991. A contribution to the pure theory of money. *Journal of Economic Theory* 53 (April): 215–35.

Kocherlakota, Narayana R. 1996. Money is memory. Research Department Staff Report 218. Federal Reserve Bank of Minneapolis.

Kocherlakota, Narayana R., and Wallace, Neil. 1996. Optimal allocations with incomplete record-keeping and no commitment. Research Department Working Paper 578. Federal Reserve Bank of Minneapolis.

Levine, David K. 1991. Asset trading mechanisms and expansionary policy. *Journal of Economic Theory* 54 (June): 148–64.

Lucas, Robert E., Jr. 1972. Expectations and the neutrality of money. *Journal of Economic Theory* 4 (April): 103–24.

___________. 1986. Principles of fiscal and monetary policy. *Journal of Monetary Economics* 17 (January): 117–34.

___________. 1996. Nobel lecture: Monetary neutrality. *Journal of Political Economy* 104 (August): 661–82.

Lucas, Robert E., Jr., and Woodford, Michael. 1993. Real effects of monetary shocks in an economy with sequential purchases. Working Paper 4250. National Bureau of Economic Research.

Mishkin, Frederic S. 1986. *The economics of money, banking, and financial markets.* Boston: Little, Brown.

Monroe, Arthur Eli. 1966. *Monetary theory before Adam Smith.* New York: Kelley.

Ostroy, Joseph Martin. 1970. Exchange as an economic activity. Ph.D. dissertation. Northwestern University.

Renero, Juan Manuel. 1994. Gresham's law type equilibria in the Kiyotaki-Wright model. Discussion Paper 9411. Instituto Technológico Autónomo de México.

___________. 1995. Welfare of alternative equilibrium paths in the Kiyotaki-Wright model. Discussion Paper 9502. Instituto Technológico Autónomo de México.

Samuelson, Paul. 1968. What classical and neoclassical monetary theory really was. Reprinted 1969. In *Monetary theory, selected readings,* ed. Robert W. Clower, pp. 170–90. New York: Penguin.

Sargent, Thomas J., and Wallace, Neil. 1983. A model of commodity money. *Journal of Monetary Economics* 12 (July): 163–87.

Shi, Shouyong. 1995. Credit and money in a search model with divisible commodities. Discussion Paper 917. Queen's Institute for Economic Research.

Smith, Adam. 1776. Reprinted 1966. *An inquiry into the nature and causes of the wealth of nations.* 2 vols. New York: Kelley.

Tobin, James. 1980. Discussion. In *Models of monetary economies,* ed. John H. Kareken and Neil Wallace, pp. 83–90. Minneapolis: Federal Reserve Bank of Minneapolis.

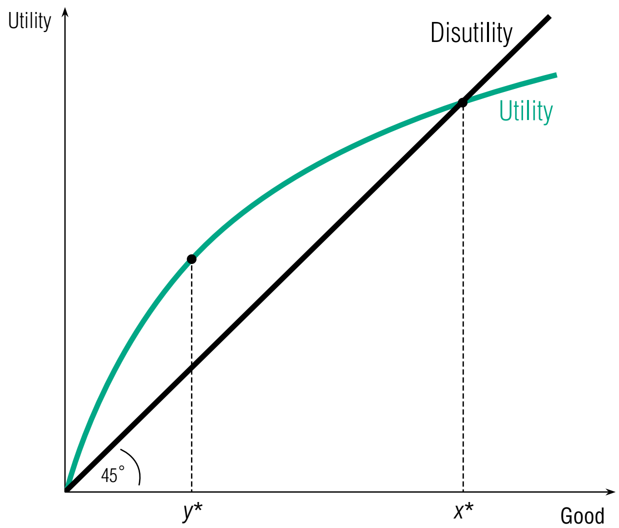
Wallace, Neil. 1996a. A dictum for monetary theory. In *Foundations of research in economics: How do economists do economics?* ed. Steven G. Medema and Warren J. Samuels, pp. 248–59. Cheltenham, U.K.: Elgar.

___________. 1996b. Short-run and long-run effects of changes in money in a random matching model. Research Department Working Paper 568. Federal Reserve Bank of Minneapolis.

Woodford, Michael. 1990. The optimum quantity of money. In *Handbook of monetary economics,* ed. Benjamin M. Friedman and Frank H. Hahn, Vol. 2, pp. 1067–1152. New York: Elsevier Science.

The Utility of Consuming
and the Disutility of Producing

At a Date



Solutions to (A3) for Small ρ_1