The Overlapping Generations Model of Fiat Money

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1. Introduction

There are two widely accepted defining characteristics of fiat money: inconvertibility and intrinsic uselessness. *Inconvertibility* means that the issuer, if there is one, does not promise to convert the money into anything else—gold or wheat, for example. *Intrinsic uselessness* means that fiat money is never wanted for its own sake; it is not legitimate to take fiat money to be an argument of anyone's utility function or of any engineering production function. Stated somewhat differently, intrinsic uselessness means that one person gives up goods (objects that appear as arguments of utility functions, directly or indirectly) for fiat money only because the person believes that someone else will subsequently give up goods for fiat money at an acceptable rate of exchange.

The argument for intrinsic uselessness is that too much is sacrificed by abandoning it. The principal way of abandoning intrinsic uselessness is to make money an argument of utility functions or engineering production functions. But this begs too many questions. Is it fiat or commodity money that appears in these functions? What if there are several fiat moneys, those of different countries? Do all appear, and if so, how? Does Robinson Crusoe have fiat money as an argument of his utility function? And what about other pieces of paper? Most economists would be unhappy with a theory of the value of shares in the XYZ Corporation that starts out making such shares an argument in utility functions. Why is it better to do this for fiat money? All of this is to say that theories that abandon intrinsic uselessness will be almost devoid of implications.

In contrast to intrinsic uselessness one might say that inconvertibility is not something to be argued about. Either the institution under investigation displays this property or it does not. But some care must be taken. As I use it, *inconvertibility* means that it is known with certainty that the issuer does not now and will never in the future stand ready to convert fiat money into a commodity. Viewed this way, fiat money systems have, I think, been rare. Specie suspensions during wars do not in general qualify. In most such instances—for example, Great Britain during the Napoleonic wars and World War I and the United States during the Civil War—convertibility was subsequently restored. That being so, it seems sensible to suppose that individuals had always attached some positive probability to such restoration, which is enough to violate what I mean by inconvertibility. But now restoration of convertibility seems unlikely. So, and perhaps for the first time, theories of inconvertible money are of practical importance.

If inconvertibility and intrinsic uselessness are taken seriously, there is an immediate and long-standing problem: the devices usually invoked to prove that an object has value in equilibrium—basically, that supply is limited and that utility is increasing in the amount consumed—cannot be used for fiat money. Since getting fiat money to have value is necessary for any nontrivial theory of it, three options seem to be available: first, one can abandon inconvertibility and intrinsic uselessness; second, one can impose legal restrictions that give fiat money value; and third, one can attempt to model explicitly the notion that fiat money facilitates exchange. For good reasons, monetary theorists are almost unanimous in pursuing the third option.

In order to pursue the notion that fiat money facilitates exchange, one must abandon the costless multilateral market clearing implicit in the Walrasian (or Arrow-Debreu) general equilibrium model. Since exchange works perfectly in that model, there can be no role for a device that is supposed to facilitate exchange. In order to get a theory of fiat money, one must generalize the Walrasian model by including in it some sort of *friction,* something that will inhibit the operation of markets. On that there is agreement.

But what sort of friction? On that there is no agreement, which is to say there is no widely accepted theory of fiat money. I will try to alter this situation by arguing that the friction in Samuelson's 1958 consumption loan model, *overlapping generations*, gives rise to the best available model of fiat money.

That this needs arguing is clear. It is now 20 years (a generation!) since Samuelson described the role of fiat money in the overlapping generations model. Yet neither he nor most economists seem to take it seriously as a model of fiat money. (See, for example, Samuelson 1968.[1]) One claim seems to be that the overlapping generations friction accounts for the store-of-value function of money only. As a consequence, it is argued, models built on it are quite misleading. In particular, the tenuousness of equilibria in which fiat money has value in models built on the overlapping generations friction is due, according to this claim, to the fact that this friction does not account for the medium-of-exchange role of money. But this claim fails to recognize that tenuousness is an implication of the two defining properties of fiat money, inconvertibility and intrinsic uselessness, and not of the overlapping generations friction.

One of the most important, although obvious, implications of inconvertibility and intrinsic uselessness is that if fiat money has value in an equilibrium, then there cannot be another asset with a rate-of-return distribution in that equilibrium which dominates that of fiat money. For suppose this were not true. Then someone gives up goods for fiat money in this equilibrium even though the person could instead buy an alternative asset that in all circumstances would give more goods subsequently. Why? Any reason that one could give would violate intrinsic uselessness.

Another implication says, in effect, that an economy can work (although maybe not so well, as we will see) without fiat money, or equivalently, without it having value. In other words, there are *nonmonetary equilibria*,

[1]Author names and years refer to the works listed at the end of this book.

equilibria in which fiat money is without value at any date.

These implications of inconvertibility and intrinsic uselessness imply that *monetary equilibria*, equilibria in which there is valued fiat money, are tenuous.

Also implicit in the definition of fiat money is the idea that in an equilibrium with valued fiat money its value exceeds its cost of production. But this rules out a competitive monetary equilibrium with free entry into the production of fiat money. Put differently, wealth is greater in the monetary equilibrium than in the nonmonetary equilibrium. Somehow this wealth must be allocated among individuals. The market cannot do that.

All these problems, if problems they be, show up in the models built on the overlapping generations friction. But all of them—absence of dominance, existence of a nonmonetary equilibrium, and wealth creation—have to be addressed by any model of fiat money that maintains inconvertibility and intrinsic uselessness. The fact that they come up in models built on the overlapping generations friction is not, therefore, a defect of that friction.

In a way, it is precisely those problems that give rise to a rich theory of fiat money. That is the message of what follows and constitutes my principal argument for taking seriously models built on the overlapping generations friction. I will show how models built on this friction can be made to confront virtually every long-standing problem in monetary economics.

The first problem I take up—and perhaps the most basic—concerns the efficiency of fiat and commodity money systems. One notion is that fiat money helps by freeing resources that would otherwise be used to produce a commodity money. In section 2, I show how models built on the overlapping generations friction give precise content to this notion. Roughly speaking, the results tie the existence of an optimal fixed-supply monetary equilibrium to the nonoptimality of the nonmonetary equilibrium.

The second problem I address concerns fiat money–financed deficits. In section 3, I show how overlapping generations models allow us to present simple public finance analyses of fiat money issue as a taxation device. Not surprisingly, fiat money issue turns out to be an excise tax.

The third problem I consider concerns paradoxical time series correlations between the quantity of fiat money and other variables. In section 4, I review, by way of a simple example, Lucas' (1972) incomplete information theory of nonstructural time series correlations. This theory shows what happens if information barriers (another friction?) force individuals to make imperfect inferences from observations on the equilibrium value of fiat money. The theory is the key building block in the current attempt to bring cyclical phenomena within the purview of ordinary economic theory. An understanding of Lucas' theory forces one to a radical reinterpretation of many of the macroeconomic time series correlations that are currently treated as structural.

The fourth problem I deal with concerns open market operations, or monetary policy narrowly conceived. The main result comes from nothing more than a careful consolidation of the balance sheets of the monetary authority and the public: without additional frictions, the portfolio of the monetary authority does not matter even for the value of fiat money. What counts is asset creation and destruction, not asset exchanges, or in other words, outside money, not inside money, or in still other words, fiscal policy, not monetary policy.

The fifth and last problem I take up concerns country-specific fiat moneys. In section 6, I summarize my work with Kareken (1978) on the relationship between national budget policies and international monetary relationships. The theme, which grows directly out of the defining properties of fiat money, is that while one may get a theory of one valued fiat money by explicit modeling of how fiat money facilitates exchange, in order to get a theory of several valued fiat moneys one must in addition invoke legal restrictions (other frictions?) that prevent one fiat money from being substituted for another. One implication is that a laissez-faire international monetary system with many national fiat moneys makes no sense.

2. Efficiency of Fiat and Commodity Money Systems

As noted above, one widely held view is that fiat money is an efficient form of money; since fiat money can be produced costlessly, there is a gain from using it instead of something else that is both costly to produce and has alternative uses. (See Friedman 1953, pp. 204–50; 1960.) While this seems plausible, several questions must be answered. Since the gain does not exist in every economy—for example, it does not exist in the world of the Walrasian general equilibrium model or in that of Robinson Crusoe—in what sort of economies does it exist? And does the gain depend on how fiat money is managed? I will provide answers to these questions in the context of a particular overlapping generations model and will then consider generalizations of it.

2.1. *Commodity Money Equilibria*

Before describing the model, a confession is in order. I suggested above that while a model may or may not have a monetary equilibrium, one in which fiat money has value, it will, in general, have a nonmonetary equilibrium, one in which fiat money has no value. The confession is that I will not distinguish between nonmonetary equilibria and commodity money equilibria. For me they are simply different names for the same thing.

While any given model may have several nonmonetary equilibria, there will in general be no basis for classifying these into two distinct classes—one called *commodity money equilibria* and the other called, say, *barter equilibria*. (This is true vacuously if the given model has a unique nonmonetary equilibrium.) Nor is it obvious that anything is to be gained from attempting to classify the nonmonetary equilibria of different models into those that, in some sense, look like commodity money equilibria and those that look like barter equilibria. True, some models may display transaction patterns such that no object is playing a special role in exchange (all objects have roughly the same transaction velocity), while other models may display patterns such that one object has a quite special role in exchange (its transaction velocity is much higher than that of any other object). But most models with friction are likely to display some intermediate pattern in which a ranking of objects by transaction velocity produces a hierarchy. I doubt that a fruitful qualitative distinction will emerge from attempting to classify models by the nature of their transaction-velocity pattern. Nor, as I hope to demonstrate, is it fruitful to group together all objects with transaction velocities greater than some number and to call the collection *money*.

2.2. *A Model With Constant Returns-to-Scale Storage*

This model, a slight variant of the one studied by Cass and Yaari (1966a), is a discrete-time, one-good economy. At any date t, the population consists of $N(t)$ young (or age 1), the members of generation t, and $N(t-1)$ old (or age 2), the members of generation $t-1$. Each young person at t maximizes $u[c^h(t)]$; $c^h(t) = [c_1^h(t), c_2^h(t)]$, where $c_j^h(t)$ is age j consumption of member h of generation t, and u is twice differentiable with convex upper contour sets. The c_j are normal goods, and u_1/u_2, the marginal rate of substitution function, approaches infinity as c_1/c_2 approaches zero and approaches zero as c_1/c_2 approaches infinity. Each old person at t maximizes $c_2^h(t-1)$.

Each young person is endowed at t with y units of the consumption good. The good may be exchanged, consumed, or stored; if $k \geq 0$ units are stored, the result is xk units of $t+1$ consumption where $x > 0$. I assume that $N(t) / N(t-1) = n > 0$ for all t.

I will study the evolution of this economy from some arbitrary initial date labeled $t=1$ for convenience. In the aggregate, the $t=1$ old, the members of generation 0, are endowed with $K(0) \geq 0$ units of the consumption good and with $M(1)$ units of fiat money.

For all t, $M(t)$, the post-transfer time t stock of money, obeys $M(t) = zM(t-1)$, $z > 0$. The time t transfer (or tax), $(z-1)M(t-1)$, is divided equally at time t among the $N(t-1)$ members of generation $t-1$. The handouts are fully anticipated and are viewed as lump-sum, as not dependent on saving or portfolio behavior.

Our two main questions about this economy are, Under what circumstances does a monetary equilibrium exist? and When it exists, under what circumstances does it improve matters?

2.2.1. *Equilibria*

Let $p(t)$ be the price of a unit of fiat money at time t in units of time t consumption. Then, letting $c(t) = [\ldots, c^h(t), \ldots]$, $k(t) = [\ldots, k^h(t), \ldots]$, $m(t) = [\ldots, m^h(t), \ldots]$ be the vectors of generation t's lifetime consumption, time t storage, and time t money purchases, respectively, an equilibrium is a sequence $[c(t-1), k(t), m(t), p(t)]$, $t=1, 2, \ldots$ that is consistent with

- $c^h(t)$, $m^h(t)$, and $k^h(t)$ being optimal for the perfect-foresight competitive choice problem of the young to be described below
- $c_2^h(0)$ being maximal for the (trivial) competitive choice problem of the current old
- $M(t) = \Sigma m^h(t)$, which, like all other unindexed summations, is over all h in generation t.

2.2.2. *The Choice Problem of the Young*

The young choose nonnegative values of $c^h(t)$, $k^h(t)$, and $m^h(t)$ to maximize $u[c^h(t)]$ subject to

$$c_1^h(t) + k^h(t) + p(t)\, m^h(t) - y \leq 0 \tag{1}$$

$$c_2^h(t) - xk^h(t) - p(t+1)\, \{m^h(t) + (z-1)\, M(t)\, / \,[N(t)]\} \leq 0 \tag{2}$$

for $p(t) \geq 0$ and $p(t+1) \geq 0$.

The necessary and sufficient conditions for an optimum are (1) and (2) at equality and

$$u_1 - \ell_1^h \leq 0 \qquad \text{with} = \text{if } c_1^h(t) > 0 \tag{3}$$

(4) $u_2 - \ell_2^h \leqslant 0$ with $=$ if $c_2^h(t) > 0$

(5) $-\ell_1^h + x\ell_2^h \leqslant 0$ with $=$ if $k^h(t) > 0$

(6) $-\ell_1^h\, p(t) + \ell_2^h\, p(t+1) \leqslant 0$ with $=$ if $m^h(t) > 0$

where ℓ_j^h is the nonnegative multiplier associated with constraint j and where, by our nonsatiety assumption about u and the boundedness and nonemptiness of the feasible $c^h(t)$, $\ell_j^h > 0$ and finite in any equilibrium.

2.2.3. *Nonmonetary Equilibrium*

By definition, $p(t) = 0$ for all t in such an equilibrium. This implies that (3)–(5) hold with equality. So, letting $v[c^h(t)] \equiv u_1/u_2$ be the marginal rate of substitution function, which by normality of the $c_j(t)$ satisfies $v_1 < 0$, $v_2 > 0$, (3)–(5) give us

(7) $$v[c^h(t)] = x.$$

Thus, there is a unique $k^h(t)$, say, k^*, for which (7) holds. [To prove this, use (1) and (2) at equality to write the $c_j^h(t)$ as functions of $k^h(t)$.]

2.2.4. *Monetary Equilibria*

By definition, $p(t) > 0$ and $m^h(t) > 0$ for some t in such an equilibrium. But by (6), this implies $p(t) > 0$ for all t. Therefore, by (3)–(6)

(8) $$v[c^h(t)] = p(t+1)\,/\,p(t) \geqslant x \qquad \text{for all } t$$

is necessary and sufficient in order that choices be optimizing in a monetary equilibrium. I can now prove

PROPOSITION 1. *$xz/n \leqslant 1$ is necessary and sufficient for the existence of at least one monetary equilibrium.*

Proof (Necessity). Suppose to the contrary that there is a monetary equilibrium and $xz/n > 1$. Then, by the rule generating $M(t)$ and the equilibrium condition, $M(t) = N(t)\,\overline{m}(t)$, where $\overline{m}(t) = \Sigma m^h(t)\,/\,N(t)$, we have for all t

(9) $$\frac{p(t+1)}{p(t)} = \frac{M(t+1)\,p(t+1)}{zM(t)\,p(t)} = \frac{N(t+1)\,\overline{m}(t+1)\,p(t+1)}{zN(t)\,\overline{m}(t)\,p(t)} = \frac{nq(t+1)}{zq(t)}$$

where $q(t) \equiv p(t)\,\overline{m}(t)$. Then, by the inequality part of (8), we have for all t

(10) $$q(t+1)\,/\,q(t) \geqslant xz/n > 1.$$

But since $q^h(t) = m^h(t)\,p(t) < y$ for all h and t, the same bound applies to $q(t)$. And no bounded $q(t)$ sequence can satisfy (10).

Proof (Sufficiency). I will prove that $xz/n \leqslant 1$ implies the existence of a monetary equilibrium with $k^h(t) = 0$ and $q^h(t) = q(t)$ for all h and $t \geqslant 1$. By (1), (2), and (8), it suffices to find a positive $q(t)$ sequence that satisfies

(11) $$v[y - q(t),\, q(t+1)\,n] = [q(t+1)\,/\,q(t)]\,(n/z) \geqslant x.$$

If there is a $q \in (0,y)$ such that $q(t) = q(t+1) = q$ satisfies the equality part of

(11), then the inequality part is implied by $xz/n \leqslant 1$. But the existence (and uniqueness) of such a q, denoted q^*, is trivial. Let $v^*(q) \equiv v(y-q, nq)$. Then v^* is continuous (strictly increasing), with $0 = \lim v^*(q)$ as $q \rightarrow 0$ from above and $\infty = \lim v^*(q)$ as $q \rightarrow y$ from below.

2.2.5. *Other Monetary Equilibria*

In the borderline case $xz/n = 1$, there exists a continuum of monetary equilibria. As is easily shown, if $xz/n = 1$, then for any $\hat{q} \in (0, q^*)$, there exists a $\hat{k} \in (0, k^*)$ such that $\hat{q}^h(t) = q$, $k^h(t) = \hat{k}$ for all $t \geqslant 1$ and h is a monetary equilibrium.

In the rest of this paper, special attention will be paid to monetary equilibria that are the analogues of the q^* and $\hat{q}$ equilibria, equilibria that I call *stationary*. But there are, in general, other monetary equilibria. Without additional restrictions on preferences, there may exist nonconstant $q(t)$ sequences that satisfy (11). Also, independent of whether preferences are further restricted, if $xz/n < 1$, then for any $q(1) \in (0, k^*)$, there exist monetary equilibria with $q^h(1) = q(1)$ for all h and $q^h(t+1) / q^h(t) = xz/n$ for all $t \geqslant 1$ and h. This is to say that if individuals in every period act on the basis of the view that per capita real money holdings decline exponentially at the rate xz/n, then there are equilibria in which this occurs.

The existence of multiple perfect-foresight or *rational expectations* equilibria has been taken by some to be a weakness of this equilibrium concept. (See Shiller 1978.) But a weakness relative to what? In models of money, actions depend on beliefs about the future. One option is to leave matters there; actions and, hence, equilibria depend on unexplained beliefs. By requiring that beliefs be tied to objective features of the environment, rational expectations equilibria allow us to focus on a subset of the equilibria consistent with all possible unexplained beliefs. The fact that this subset turns out to contain more than one equilibrium does not provide an argument for abandoning the rational expectations equilibrium concept. Instead, it suggests that we look for a principle that justifies focusing on a subset of the rational expectations equilibria. We know already that we will not be able to justify focusing on equilibria not in the subset of rational expectations equilibria.

One argument for focusing on the k^*, q^*, and $\hat{q}$ equilibria starts out by requiring that equilibrium values depend only on relevant state variables, or, equivalently, by requiring that agents forecast on this basis. In the model we are studying, each member of generation t for $t \geqslant 1$ must forecast $p(t+1)$. But since the evolution of the money supply and the population are known, it is enough to forecast $q(t+1)$. If each young person acting like a competitor responds to the calling out of an arbitrary current price $p(t) \geqslant 0$ with a money demand based on a forecast of $p(t+1)$ that satisfies $q(t+1) = q(t)$, then such demands are consistent with the k^*, q^*, and $\hat{q}$ equilibria only. This forecasting scheme must be justified by some appeal to relevant state variables; since the relevant physical environment for each young person in this economy does not depend on when the young person appears, the person's real portfolio behavior ought not to depend on time. In monetary equilibria other than the $\hat{q}$ and q^* equilibria, the variable time plays a significant role.

2.2.6. *A Stability Result*

I will now present for this model a sort of stability result adapted from the argument presented by Lucas and Prescott (1974).

First note that without additional assumptions, the equality part of (11) is equivalent to

(12) $$q(t) = H[q(t+1)]$$

where H, a continuous function, is defined on $(0,\infty)$, is such that $0 < H < y$, and has a unique fixed point, denoted $\bar{q}$. (The range of H is limited to the shaded area in Figure 1.) It is an implication that $H'(\bar{q}) < 1$. For the stability argument I assume $\ell < H' < 1$ for some $\ell \in (-\infty, 1)$, which aside from boundedness is to impose globally the restriction on H' that necessarily holds at the fixed point.

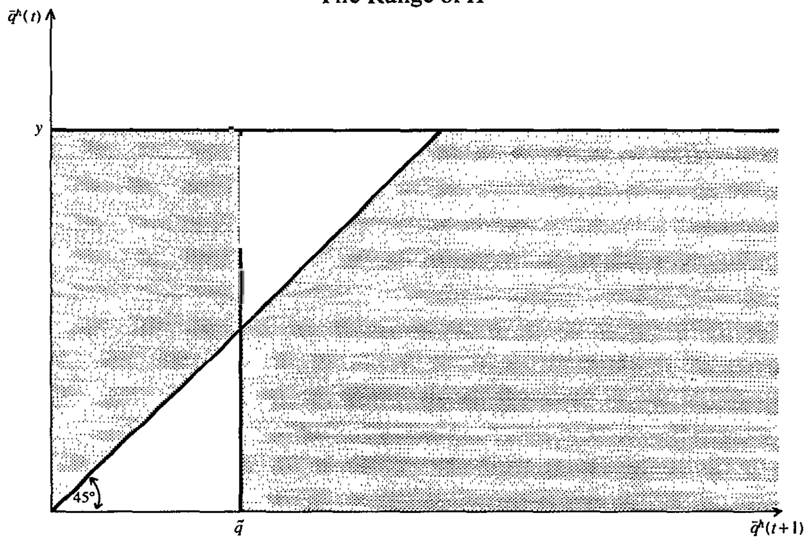
For the forecasting scheme, I assume that every member of generation 1 acts on the basis of some positive forecast of $p(2)$, denoted $\bar{p}(2)$, and that every member of generation $t \geqslant 2$ acts on the basis of a point forecast $\bar{p}(t+1)$ given by

(13) $$\bar{p}(t+1) = [\lambda\, \bar{p}(t) + (1-\lambda)\, p(t)]\,(n/z), \quad \lambda \in [0,1).$$

Here generation $t \geqslant 2$ forms a forecast of $p(t+1)$ by taking a weighted average of the previous generation's forecast of $p(t)$ and the realization of $p(t)$ and "updating" it. The "updating" is accomplished by multiplying by n/z. Note that the scheme discussed above is the special case $\lambda = 0$. In the Appendix I prove

PROPOSITION 2. *If forecasts are given by (13) for some $\bar{p}(2) > 0$ and H [see (12)] satisfies $H' \in (\ell, 1)$ for some $\ell \in (-\infty, 1)$, then*

Figure 1
The Range of H



a. *If $xz/n < 1$, then $\lim q^h(t) = q^*$*

b. *If $xz/n = 1$ and $\bar{q}(2) \geqslant q^*$, then $\lim q^h(t) = q^*$*

c. *If $xz/n = 1$ and $\bar{q}(2) < q^*$, then $q^h(t) = \bar{q}(2)$*

d. *If $xz/n > 1$, then $\lim q^h(t) = 0$*

where $\bar{q}(2) = \bar{p}(2)\, M(2) \,/\, N(2)$.

This proposition establishes that the q^* and $\hat{q}$ equilibria—when they exist—and the k^* equilibrium—when the q^* and $\hat{q}$ equilibria do not exist—are in a sense robust with regard to mistakes about the future.

It is worth emphasizing, though, that while the proposition would no doubt hold for some ways of generalizing of (13), the "updating" by n/z seems essential. Such "updating" must be justified by appealing to relevant state variables. Thus, this stability argument cannot be used, for example, to dispose of equilibria with $q^h(t+1) \,/\, q^h(t) = xz/n$ when $xz/n < 1$. For if (n/z) in (13) is replaced by x, then the same sort of stability argument could no doubt be made for those equilibria.

2.2.7. *Optimality*

Letting $C_j(t)$ be total age j consumption of members of generation t and $K(t) \geqslant 0$ be the total of time t output that is stored, a consumption allocation—$c_2(0)$, $c(1)$, $c(2)$, ..., $c(t)$, ...—is *feasible* if for all $t \geqslant 1$ there exists $K(t) \geqslant 0$ that satisfies

$$C_1(t) + K(t) + C_2(t-1) \leqslant N(t)y + xK(t-1) \tag{14}$$

with $K(0)$ given by initial conditions.

An allocation with a bar ($^-$) over it is *Pareto superior* to one with a caret ($\hat{}$) if $\bar{c}_2^h(0) \geqslant \hat{c}_2^h(0)$ and $u[\bar{c}^h(t)] \geqslant u[\hat{c}^h(t)]$ for all h and all $t \geqslant 1$ with strict inequality somewhere. An allocation with a caret is *Pareto optimal* if there does not exist a feasible allocation that is Pareto superior to it.

Our question about the gain from using fiat money is answered in part by the following propositions.

PROPOSITION 3. *If $x > n$, then any equilibrium allocation is optimal.*

PROPOSITION 4. *If $x \leqslant n$, then the k^* (the nonmonetary) and the $\hat{q}$ equilibria are nonoptimal.*

PROPOSITION 5. *If $xz/n \leqslant 1$ and $z \leqslant 1$, then the q^* equilibrium is optimal.*

PROPOSITION 6. *If $xz/n \leqslant 1$ and $z > 1$, then the q^* equilibrium is nonoptimal.*

The proofs of Propositions 3 and 5 are given in the Appendix.

Proof of Proposition 4. Let $(\bar{c}_1, \bar{c}_2)$ and $\bar{k} > 0$ be the equilibrium lifetime consumption vector and storage of each member of generation $t \geqslant 1$ in any given k^* or $\hat{q}$ equilibrium. By feasibility [see (14)] these quantities satisfy

$$\bar{c}_1 + \bar{c}_2/n \leqslant y + (x/n - 1)k.$$

Since $x \leqslant n$, it follows from (14) that the same consumption allocation for members of generation $t \geqslant 1$ is feasible with $K(t) = 0$ for all $t \geqslant 1$. But this makes it possible to give more to members of generation 0 at $t = 1$ because in any k^* or $\hat{q}$ equilibrium, total consumption of generation 0 at $t = 1$ satisfies

$$N(1)\,\bar{c}_1 + N(1)\,\bar{k} + C_2(0) = N(1)y + xK(0)$$

while under the proposal it satisfies the same equation with $\bar{k}$ replaced by zero.

Q.E.D.

Proof of Proposition 6. By (14), if $K(t) = 0$ for all $t \geqslant 1$, the class of feasible consumption allocations such that $c^h(t) = (c_1, c_2)$, a constant for all h and $t \geqslant 1$, is given by

$$c_1 + c_2/n \leqslant y. \tag{15}$$

It follows that $c^h(t)$ for all h and $t \geqslant 1$ in any q^* equilibrium satisfies (15) and with equality. Denote $c^h(t)$ in this equilibrium by (c_1^*, c_2^*). It is also the case [see (1) and (2)] that (c_1^*, c_2^*) is preference-maximizing in the set

$$c_1 + c_2\,(z/n) \leqslant y + (z-1)\,q^*. \tag{16}$$

In Figure 2, let $(\hat{c}_1, \hat{c}_2)$ be the preference-maximizing point that satisfies (15). Transitivity and the fact that the boundary of (16) is steeper than that of (15) imply that (c_1^*, c_2^*) lies northwest of $(\hat{c}_1, \hat{c}_2)$. By revealed preference, then, $(\hat{c}_1, \hat{c}_2)$ is preferred to (c_1^*, c_2^*). And since $c^h(t) = (\hat{c}_1, \hat{c}_2)$ for all h and $t \geqslant 1$ is feasible with $K(t) = 0$ for all $t \geqslant 1$, it is feasible under the caret allocation to have total consumption of generation 0 at $t = 1$ satisfy

$$\hat{C}_2(0) = xK(0) + N(1)\,(y-\hat{c}_1) > xK(0) + N(1)\,(y-c_1^*) = C_2^*(0)$$

where $C_2^*(0)$ is total generation 0 consumption at $t = 1$ under the q^* equilibrium.

Q.E.D.

Propositions 3–6 imply no connection between the path of the value of fiat money in an equilibrium and the optimality of that equilibrium. Thus there may be an optimal monetary equilibrium with a declining value of fiat money—for example, if $x < n < z \leqslant 1$—and there may be a nonoptimal monetary equilibrium with an unchanging value of fiat money—for example, if $z = n > 1 > x$. What counts is the path of the quantity of fiat money.

Propositions 3–6 suggest that the quantity of fiat money should not be increased. For those propositions imply that if $z \leqslant 1$, then an optimal monetary equilibrium exists whenever the nonmonetary is nonoptimal. If we limit consideration to $z = 1$, a fixed money supply, then the converse is also true.

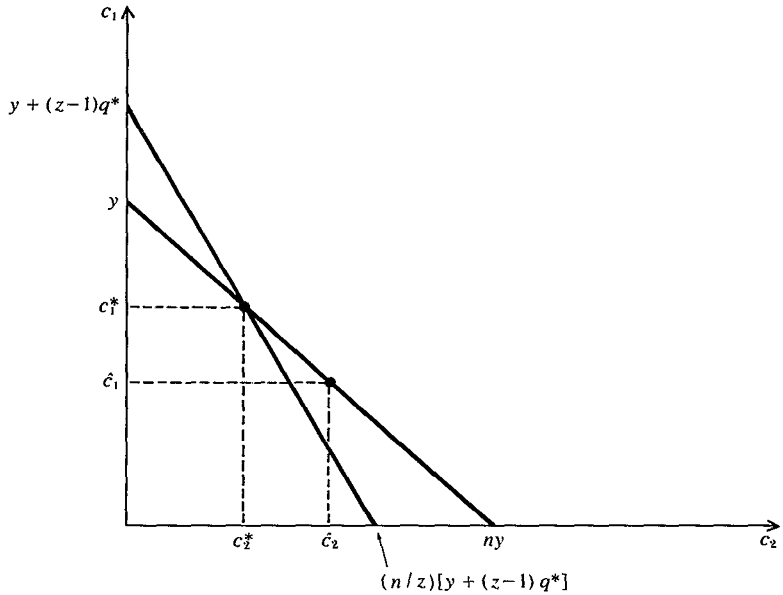
But several comments are in order. First, note that we have studied changes in fiat money brought about in a particular way. These changes satisfy

$$-\tau(t) = p(t)\,[M(t) - M(t-1)] = p(t)\,(z-1)\,M(t-1)$$

where $\tau(t)$ is the national income accounts definition of taxes minus transfers in real terms and where the tax or transfer is lump-sum.

So this conclusion holds for fiat money supply increases that come about

Figure 2
The Nonoptimality of $z > 1$



by way of deficits. They are most definitely not to be confused with changes that come about through open market operations, a class of policies discussed in section 5. The conclusion also holds only for purposeless deficits, a point taken up in section 3.

That the tax or transfers are viewed by individuals as unrelated to their saving and portfolio choices is also critical. To take an extreme example, if the scheme is instead one in which each person knows that her or his money purchases when young, $m^h(t)$, say, will be augmented by $(z-1)\, m^h(t)$ when old, then, as is well known, the equilibrium consumption allocation does not depend on z.

It must also be emphasized that we have studied only very special time paths of $M(t)$. In a sense, only the limiting behavior of the supply of fiat money is crucial. For, still considering the same transfer mechanism, suppose

$$M(t) = \begin{cases} z_1\, M(t-1) & t = 1, 2, \ldots, T \\ z_2\, M(t+1) & t \geqslant T+1. \end{cases}$$

Then it is easily shown that Propositions 5 and 6 hold with z replaced by z_2 and q^* replaced by a monetary equilibrium path with $q^h(t) = q^*$ for all h and $t \geqslant T$. In other words, both the existence and optimality of monetary equilibria depend only on the limiting behavior of the supply of fiat money. (The equilibrium consumption allocation does, however, depend on z_1.)

Finally, this model does not deal kindly with the prescription that money ought to be managed so that its rate of return is equal to that on real capital. In most models, this is not even a well-defined prescription since the equilibrium rate of return on capital depends on the monetary policy followed.[2] In this model, the technology is such that this prescription is well defined; namely, set z so that $xz/n = 1$. But if $x < n$, then such a policy guarantees that any equilibrium is nonoptimal.

2.3. *Absence of a Dominating Asset and the Sign of the Real Rate of Interest*

In the model so far analyzed, individuals hold a single asset except in the borderline case $xz/n = 1$. In any case, individuals do not diversify in a determinate way. It is important to note that the basic message of Propositions 1–6 carries over to other models.

Thus suppose that the gross return on storage x is random in the following way. The gross return on the consumption goods stored from t to $t+1$ is $x(t) > 0$ where $x(t)$ is a drawing from a probability distribution. For some utility functions, probability distributions, and values of n and z, there exist monetary equilibria with equilibrium portfolios that are diversified in a determinate way. (See Bryant and Wallace 1979b.) More to the point, though, versions of Propositions 1–6 can be expected to hold.[3] And, for any z and n, if the probability distribution of returns to storage is too favorable, then there will not exist a monetary equilibrium. Thus the absence of dominance requirement still obtains.

Since this requirement is foreign to so many so-called models of money, I want to comment on it in some detail. And to simplify the discussion, I will abstract from growth.

Economists are used to working with models in which there are real assets with a positive net yield in equilibrium. Put differently, most economists seem automatically to assume a positive real rate of interest implied by the presence of something like the "land" of the Hecksher-Ohlin model (Henry George land)—a uniform, productive, nonreproducible form of capital.[4] But real land is quite different from the land of the Hecksher-Ohlin model. If it were not, why have titles to physical quantities of land rarely if ever served as commodity money? (And why does it prove so difficult to implement Henry George's tax?) Indeed, one suspects that the transaction velocity of real land is not very high (relative, say, to that of shares in GM). The point,

[2]Tobin's (1965) is one of the first models with this property. The model described in the text could easily be converted to one in which the equilibrium rate of return on storage of the consumption good depends on the monetary policy followed. One need only assume that x depends on the aggregate amount stored.

[3]A model that allows for intergeneration risk sharing via markets arises from assuming that generation $t+1$ appears prior to the realization of $x(t)$. One that precludes such risk sharing arises from assuming that the realization of $x(t)$ occurs after generation $t-1$ disappears and before generation $t+1$ appears. Moreover, since in the latter instance, the young of generation t maximize expected utility conditional on $x(t)$ while in the former they maximize unconditionally, the definition of Pareto superiority should take this difference into account. (See Muench 1977.)

I suspect that Muench's result in his appendix on the nonoptimality of nonstochastic growth rate rules according to his conditional Pareto optimality criterion is due to his scheme for allocating old people across markets. This scheme is not necessarily part of Lucas' model.

[4]For a detailed model of traded land of this sort, see Kareken and Wallace 1977.

of course, is that real land is nonuniform, is reproducible, and may not be productive. To abstract from these features for some purposes may be harmless. To abstract from them in thinking about fiat money is probably disastrous.

Also, one should not be misled by the role of physical appreciation in the model analyzed above. In that model $x > n$ is enough to drive out fixed-supply fiat money. But this does not mean that the existence of any good that appreciates physically would do. The $x > n$ results found above depend on all aspects of the model—that there is only one good, that the population and its endowment of that good are nonrandom, and so on. Physical appreciation does not imply dominance of fixed-supply fiat money if there are other sources of supply and demand disturbances that in a multigood model make the price of the physically appreciating good random. Thus, for example, the possibility of gold discoveries makes gold a less perfect commodity money than would otherwise be the case.

All of this is not to say that the world we live in, without various legal restrictions, necessarily has room for valued fixed-supply fiat money. It is meant to suggest that the possibility cannot be easily dismissed by casual empiricism that appeals to the existence of a positive (net of growth) real rate of interest given by the technology. Hahn dismisses the possibility in his discussion of the overlapping generations model when he says (1973a, p. 232), "But money is not the only means of storage nor the only costless means (recall Keynes's remarks on land)."

2.4. *Private Borrowing and Lending and So-Called Inside Money*

Models built on the overlapping generations friction imply a sharp distinction between what used to be called *inside money* and *outside money*. I will present one aspect of this distinction here by considering a version of the model we have been studying that has room for private borrowing and lending. Another aspect of this distinction will come up in the discussion of open market operations.

I will proceed by describing an example. Let $n = z = 1$ (an unchanging population and money supply), $x = 0$ (no storage), and let generation t consist of two groups: $N_1(t) = \alpha N$ and $N_2(t) = [1-\alpha]\, N$, $0 < \alpha < 1$, where $h \in N_1(t)$ is endowed with y units of the consumption good when young and $h \in N_2(t)$ is endowed with y units when old. For all h and $t \geq 1$, assume that $u[c^h(t)] = c_1^h(t)c_2^h(t)$.

A quick route to the study of equilibria for this economy analogous to the k^*, q^*, and $\hat{q}$ equilibria is to find equilibria consistent with there being a single, unchanging equilibrium gross rate of return on saving denoted R. In such an equilibrium we have, for the logarithmic utility function,

$$v(y-s_1, Rs_1) = Rs_1 / (y-s_1) = R \tag{17}$$

$$v(-s_2, y + Rs_2) = (y + Rs_2) / (-s_2) = R \tag{18}$$

where $v = u_1 / u_2$ and s_i is real saving of each member of group N_i.

A nonmonetary equilibrium of this type must satisfy (17), (18), and $\alpha s_1 + (1-\alpha)\, s_2 = 0$, the last condition being the nonmonetary equilibrium condition for this economy: net saving must be zero. Clearly the solution is unique and implies $R = (1-\alpha) / \alpha$ and

$$c^h(t) = \begin{cases} [y/2, y(1-\alpha)/2\alpha] & \text{if } h \in N_1 \\ [\alpha y/2(1-\alpha), y/2] & \text{if } h \in N_2. \end{cases}$$

The proofs of Propositions 3 and 4 imply that this equilibrium is optimal if $\alpha \leq 1/2$ and nonoptimal otherwise. Those proofs apply because in terms of resources and preferences this economy is a special case of the one studied in those proofs.

A monetary equilibrium consistent with an unchanging rate of return must satisfy (17), (18), $R = 1$, and $\alpha s_1 + (1-\alpha) s_2 > 0$, the last condition being a positive net saving requirement. Since $s_1 = -s_2 = y/2$ at $R = 1$, this exists if and only if $\alpha > 1/2$. Moreover, $p(t)M$, the equilibrium aggregate real value of fiat money is $N[\alpha s_1 + (1-\alpha) s_2] = y(2\alpha-1) N/2$. Clearly, when this equilibrium exists, it is optimal.

Now, suppose that there are K such economies identical except that $1/2 < \alpha_1 < \alpha_2 < \ldots < \alpha_K < 1$. In particular, let the fixed supply of fiat money be the same in all these economies.

Viewed as a cross section, then, the price of fiat money varies directly with α_K. And since M/N, the per capita supply of fiat money, is the same for all these economies, it necessarily bears no relationship to the equilibrium value of $p(t)$ in the different economies. But if for some reason we try to find some nominal asset total that is related to $p(t)$ across these economies, then we will succeed. In a monetary equilibrium, members of group N_1 in the aggregate in economy k hold assets with real value of $\alpha_k Ny/2$. Therefore, the nominal value of their assets is $M/(2-\alpha_k^{-1})$, real assets divided by the price of fiat money in economy k. This nominal asset total is decreasing in α_k, as is $1/p(t)$, the price level.

It is also the case that similar relationships can be generated for the time paths of asset totals and the price level in a single economy by studying economies in which $\alpha(t)$ varies through time.

But what is to be inferred from such relationships? The economy just examined is one in which members of group 1, the only holders of assets, are indifferent between holding fiat money and holding promises of members of group 2. (It is, of course, easy to destroy this indifference. One way is to make the endowment of group 2 random.) But so what? That does not justify aggregating the two into something called M_i and ignoring the distinction between fiat money and the debt of group 2. Since versions of Propositions 3–6 apply to this class of economies, the distinction is critical. The point, of course, is that α influences the demand for fiat money in a way not all that different from the influence exerted by $N(t)$ or x in the model of section 2.2. Even though variations in the demand for fiat money affect the value of a fixed stock of it, no response in the form of a value of $z \neq 1$ is called for.

3. Fiat Money Issue as Taxation

One of the basic propositions of public finance asserts that if endowment taxation is costless, then it is best. I will begin by establishing a version of this proposition for a no-storage (x=0) version of the model studied in section 2.2.

PROPOSITION 7. *If $N(t)\beta y$ for $0 < \beta < 1$ can be raised each period through fiat money issue, then with costless taxation of endowments there exists a monetary equilibrium with a constant money supply that is Pareto superior to* *any stationary equilibrium with $N(t)\beta y$ raised through fiat money issue.*

Proof. Let $(\hat{c}_1, \hat{c}_2)$ be the q^* equilibrium allocation with a fixed money supply and with each young person subject to endowment taxation of βy. It follows from (1) and (2) that $(\hat{c}_1, \hat{c}_2)$ satisfies

$$(19) \qquad c_1 + c_2/n \leqslant (1-\beta)y$$

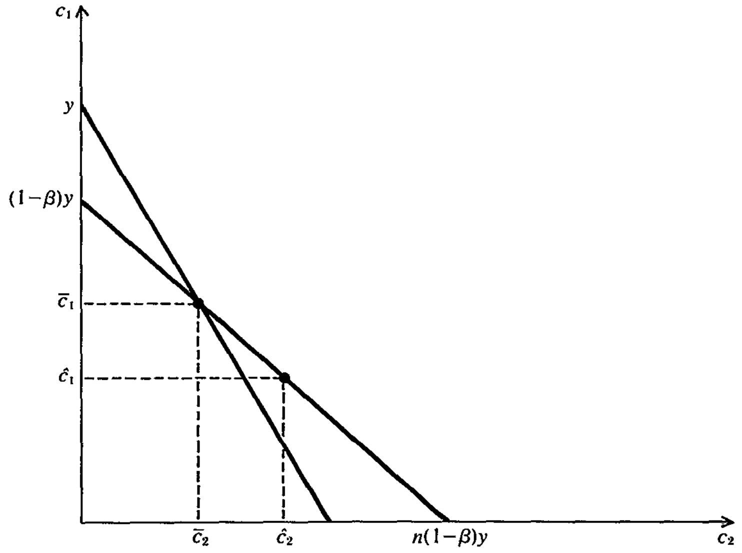
with equality and is preference-maximizing in this set. Let $(\bar{c}_1, \bar{c}_2)$ be any stationary equilibrium with $N(t)\beta y$ raised through fiat money issue. [Bryant and Wallace (1979a) prove that the existence of any equilibrium with $N(t)\beta y$ raised through fiat money issue implies the existence of a stationary equilibrium in which this occurs.] By feasibility $(\bar{c}_1, \bar{c}_2)$ also satisfies (19). And being an equilibrium, it satisfies (19) with equality and, by (1) and (2), is preference-maximizing in the set

$$(20) \qquad c_1 + c_2/R \leqslant y$$

where R equals $p(t+1)/p(t)$, a constant in a stationary equilibrium.

But these facts imply that $(\hat{c}_1, \hat{c}_2)$ does not satisfy (20), which is to say that $(\bar{c}_1, \bar{c}_2)$ lies northwest of $(\hat{c}_1, \hat{c}_2)$. (See Figure 3.) If this were not true, transitivity or smoothness of preferences would be violated. It follows that the caret allocation is Pareto superior to the bar allocation because $(\hat{c}_1, \hat{c}_2)$ is preferred to $(\bar{c}_1, \bar{c}_2)$ by all members of all generations $t \geqslant 1$ and $\hat{c}_1 < \bar{c}_1$ implies

Figure 3
Costless Endowment Taxation
vs. a Fiat Money–Financed Deficit



that the current old get more under the caret allocation than under the bar allocation.

Q.E.D.

Note, of course, that there may not be such a bar allocation and that there may be more than one. (For an excise tax on any commodity, there may be no tax that raises a given revenue or there may be many.) Note also that although in this simple model fiat money issue is a tax on saving, the nonoptimality of such taxation does not require that saving depend on the rate of return in any systematic way. In particular, the proof holds for a Cobb-Douglas utility function which implies no dependence of saving on the rate of return. Nor does the nonoptimality depend on there being inflation in the sense of a falling value of fiat money. But if there is an equilibrium bar allocation, then the equilibrium value of R is less than n. Thus, the growth-corrected rate of return must be negative. If it were not, the government could not succeed in financing its deficit.

Of course, in general, fiat money issue is not a tax on all saving. It is a tax on saving in the form of money. But it is important to emphasize that the equilibrium rate-of-return distribution on the equilibrium portfolio does depend on the magnitude of the fiat money–financed deficit. This is true, for example, in a model with a risky real asset that gives rise to diversified equilibrium portfolios and is true in a model in which the rate of return on money is random because the size of each generation is random.

In all these models, the real rate-of-return distribution faced by individuals in equilibrium is less favorable the greater the fiat money–financed deficit. Many economists seem to ignore this aspect of inflation because of their unfounded attachment to Irving Fisher's theory of nominal interest rates. [According to this theory, (most?) real rates of return do not depend on the magnitude of anticipated inflation.] The attachment to Fisher's theory of nominal interest rates accounts for why economists seem to have a hard time describing the distortions created by anticipated inflation. The models under consideration here imply that the higher the fiat money–financed deficit, the less favorable the terms of trade—in general, a distribution—at which present income can be converted into future income. This seems to be what most citizens perceive to be the cost of anticipated inflation.

Of course, Proposition 7 is only the start of an analysis of fiat money issue as a taxation device. The modern public finance literature often starts by ruling out endowment taxation.[5] And, as has often been noticed, if all taxes are distorting, then the optimal tax structure may involve taxation through fiat money issue.

A simple way to amend the model of section 2.2 to allow for this possibility is to impose costs of raising revenue through endowment taxation. Thus $(\hat{c}_1,\hat{c}_2)$ in Figure 3 is not feasible if it is assumed that $(\beta+g)y$ must be turned over by each young person in order that the government get βy—$gy > 0$ being enforcement costs. It follows, then, that the fixed-supply–monetary-equilibrium allocation satisfies (19) with strict inequality and that no qualitative comparison between endowment taxation and fiat money–issue taxation is possible. Indeed, it is easy to make assumptions that guarantee that the optimal tax structure involves some of both forms of taxation.

[5]See Helpman and Sadka 1979 for an analysis of fiat money issue as a taxation device when only commodity taxes are allowed.

4. Lucas' Incomplete Information Model

So far nothing has been said about business cycles. Among the aspects of business cycles that call for explanation are time series relationships between, on the one hand, nominal aggregate demand variables like the deficit and, on the other hand, real variables like total output. These relationships appear paradoxical from at least two points of view. First, ordinary market-clearing models with behavior generated by optimization free from various kinds of money illusion do not seem to generate them. Second, such relationships do not seem to be invariant; they do not hold in cross sections where the observations are averages over time of the same variables for different countries, and they do not seem to persist over time. Lucas' (1972) model shows how relationships with these properties can arise if ordinary market-clearing models are altered so that individuals are faced with incomplete information in a way that induces them to make inferences from observations on the value of money. I will present a very simple version of Lucas' model.

4.1. *The Model*

The model is a variant of that in section 2.2. The size of each generation is random: $N(t) = N_i > 0$ with probability $\pi_i > 0$, $i=1, 2$. The tax-transfer scheme is that of section 2.2 except that it too is random: $M(t) = z(t)M(t-1)$ where $z(t) = z_j > 0$ with probability $\theta_j > 0, j=1, 2$. $N(t)$ and $z(t)$ are independent of one another. Moreover, $N_1/N_2 = z_1/z_2 < 1$, the equality being a very special assumption, the need for which is discussed below. There is a common von Neumann–Morgenstern utility function $u[c^h(t)]$, the expected value of which is maximized by each young person. And, as above, each young person is endowed with y units of the single consumption good when young. Storage is ruled out.

Two versions of this model will be studied. One is a *complete* information version. In it, the young at time t choose money holdings knowing $M(t-1)$, $z(t)$, and $N(t)$; that is, they know the money supply, $M(t)$, and the size of their generation, a demand variable. They also know the distributions of $z(t+1)$ and $N(t+1)$. In the *incomplete* information version, the young at t choose money holdings knowing only $M(t-1)$ and the distributions of $z(t)$, $z(t+1)$, $N(t)$, and $N(t+1)$. They do, however, draw inferences about $z(t)$ and $N(t)$ from observing $p(t)$, the time t value of money.

It is convenient to begin by rewriting constraints (1) and (2) as

$$c_1^h(t) \leq y - q^h(t) \tag{21}$$

$$c_2^h(t) \leq \{q^h(t)+\bar{q}(t)[z(t+1)-1]\}p(t+1)/p(t) \tag{22}$$

where $q^h(t) = p(t)m^h(t)$ is the choice variable, $\bar{q}(t) = p(t)M(t)/N(t)$ is per capita real money holdings, which the individual treats parametrically, and where, by the definition of $\bar{q}(t)$,

$$p(t+1)/p(t) = N(t+1)\bar{q}(t+1)/N(t)\bar{q}(t)z(t+1). \tag{23}$$

In order to proceed, the distribution of $c_2(t)$, which depends on the information restrictions, must be specified. The procedure to be followed is, first, to in effect make a guess about certain properties of the equilibrium $c_2(t)$ distribution, and, second, to show that there is in fact an equilibrium satisfying these properties.

4.1.1. *Complete Information*

Here the guess is that there is an equilibrium in which for each value of $N(t)$ there is at most one value of $\bar{q}(t)$, that is, an equilibrium such that $\bar{q}(t) = q_i$ if $N(t) = N_i$.

Now if $N(t) = N_i$ and individuals forecast on the basis of $\bar{q}(t) = q_i$, then by (23) the distribution of $p(t+1)/p(t)$ is given by $p(t+1)/p(t) = N_j q_j / N_i q_i z_k$ with probability $\pi_j \theta_k$, there being four possible values of $p(t+1)/p(t)$. This implies a distribution for the right-hand side of (22), namely,

$$c_{2i}(j,k) \leq [q_i^h(t) + q_i(z_K - 1)] N_j q_j / N_i q_i z_k \tag{24}$$

with probability $\pi_j \theta_k$ where $c_{2i}(j,k)$ is second-period consumption if $N(t) = N_i$, $N(t+1) = N_j$, and $z(t+1) = z_k$. Given $N(t) = N_i$, for each value of $q_i^h(t)$, the choice variable, there are four possible values of $c_{2i}(j,k)$ as j and k each range over 1 and 2.

Thus, if $N(t) = N_i$, $q_i^h(t)$ is chosen to maximize $\Sigma_j \Sigma_k \pi_j \theta_k u[c_{1i}, c_{2i}(j,k)]$ subject to (21) and (24) with q_i and N_i known and treated parametrically. It follows that for $i=1$, 2 the optimal value of $q_i^h(t)$ satisfies (21) and (24) with equality and

$$0 = \sum_j \sum_k \pi_j \theta_k \{u_1[c_{1i}, c_{2i}(j,k)] - u_2[c_{1i}, c_{2i}(j,k)] N_j q_j / N_i q_i z_k\}. \tag{25}$$

Since all members of generation t are identical, for equilibrium we must have $q_i^h(t) = q_i$ for $i=1, 2$. Therefore, for each i the equilibrium q_i satisfies

$$0 = \sum_j \sum_k \pi_j \theta_k \{u_1(y - q_i, q_j N_j / N_i) - u_2(y - q_i, q_j N_j / N_i) N_j q_j / N_i q_i z_k\} \tag{26}$$

giving us a pair of equations in q_1 and q_2. Letting $\eta = \Sigma_k \theta_k / z_k$, the mean of z_k^{-1}, we may rewrite (26) as

$$\begin{aligned} 0 = {} & \sum_j [\pi_j u_1(y - q_i, q_j N_j / N_i)] \\ & - \eta \sum_j [\pi_j u_2(y - q_i, q_j N_j / N_i) N_j q_j / N_i q_i], \quad i=1,2. \end{aligned} \tag{27}$$

Notice that for any equilibrium of this kind, η is the only aspect of the distribution of $z(t)$ that matters for real variables and, hence, for expected utility. It follows that in an equilibrium of this kind, there is no correlation between $z(t)$ and real variables.

4.1.2. *Incomplete Information*

Here the young choose $q^h(t)$ knowing only $M(t-1)$ and the current value of fiat money, $p(t)$. The surmise is that there is an equilibrium in which for given $M(t-1)$, $p(t)$ and $N(t)/z(t)$ are in one-to-one correspondence with $p(t)$ strictly increasing in $N(t)/z(t)$. Note that $N(t)/z(t)$ may be thought of as a measure of the demand for money at t relative to the supply at t and that for our specification there are three distinct values of $N(t)/z(t)$.

In such an equilibrium, knowledge of $p(t)$ implies knowledge of $N(t)/z(t)$. And for two of the possible values of $N(t)/z(t)$—namely, N_1/z_2 and N_2/z_1—knowledge of the ratio $N(t)/z(t)$ implies knowledge of both $N(t)$ and $z(t)$. But because $N_1/N_2 = z_1/z_2$, if $N(t)/z(t) = N_i/z_i$, its intermediate value, then either

$N(t) = N_1$ and $z(t) = z_1$ or $N(t) = N_2$ and $z(t) = z_2$. In other words, if $p(t)$ takes on its intermediate value, then either both demand and supply are low or both are high. This is what gives rise to more uncertainty in this version than in the complete information version.[6]

The hypothesized correspondence between $p(t)$ and $N(t)/z(t)$ implies that an observation on $p(t)$ is equivalent to an observation on $\bar{q}(t)$ since $\bar{q}(t) = p(t)M(t-1)[z(t)/N(t)]$ and $M(t-1)$ is known. We further hypothesize that the equilibrium is such that $\bar{q}(t)$ is a function of $N(t)/z(t)$, namely, that $\bar{q}(t) = q_{ij}$ if $N(t) = N_i$ and $z(t) = z_j$ with $q_{11} = q_{22}$. This is equivalent to hypothesizing that the equilibrium sample space for the price level, $1/p(t)$, is proportional to the known pretransfer stock of money, $M(t-1)$.

The distribution of $c_2(t)$ conditional on $N(t)/z(t)$ can now be described. If $N(t)/z(t) = N_i/z_j$ with $i \neq j$, then for all k and r

$$c_{2ij}(k,r) \leq [q_{ij}^h(t)+q_{ij}(z_r-1)]N_k q_{kr}/N_i q_{ij} z_r \tag{28}$$

with probability $\pi_k\theta_r$. Aside from different subscripting, this is the same as (25). Thus, if $i \neq j$, then for each value of the choice variable, $q^h(t)$, there are four possible values of second-period consumption. If $N(t)/z(t) = N_i/z_i$, then for all i, k, r

$$c_{2ii}(k,r) \leq [q_{ii}^h(t)+q_{ii}(z_r-1)]N_k q_{kr}/N_i q_{ii} z_r \tag{29}$$

with probability $\rho_i\pi_k\theta_r$, where $\rho_i = \pi_i\theta_i/(\pi_1\theta_1+\pi_2\theta_2)$ is the probability that $N(t) = N_i$ given that $[N(t),z(t)]$ is either (N_1,z_1) or (N_2,z_2). In this case, the value of $N(t) = N_i$ cannot be identified so there are eight possible values for second-period consumption for each value of the choice variable, $q_{ii}^h(t)$.

Therefore, if $N(t)/z(t) = N_i/z_j$ with $i \neq j$, then expected utility is $\Sigma_k\Sigma_r\pi_k\theta_r u[c_{1ij},c_{2ij}(k,r)]$. This is maximized by choice of $q_{ij}^h(t)$ subject to (21) and (28). Thus for $(i,j) = (1,2)$ and $(i,j) = (2,1)$, the optimal choice satisfies (21) and (28) with equality and

$$0 = \sum_k \sum_r \pi_k\theta_r\{u_1[c_{1ij},c_{2ij}(k,r)]-u_2[c_{1ij},c_{2ij}(k,r)]N_k q_{kr}/N_i q_{ij} z_r\}. \tag{30}$$

If $N(t)/z(t) = N_i/z_i$, then expected utility is $\Sigma_i\Sigma_k\Sigma_r\rho_i\pi_k\theta_r u[c_{1ii},c_{2ii}(k,r)]$. This is maximized by choice of $q_{ii}^h(t)$ subject to (21) and (29). The optimal choice satisfies the constraints with equality and

$$0 = \sum_i \sum_k \sum_r \rho_i\pi_k\theta_r\{u_1[c_{1ii},c_{2ii}(k,r)] - u_2[c_{1ii},c_{2ii}(k,r)]N_k q_{kr}/N_i q_{ii} z_r\}. \tag{31}$$

Since for equilibrium we must have $q_{ij}^h = q_{ij}$, (30) and the relevant constraints at equality give us two equilibrium conditions: for $i \neq j$

$$0 = \sum_k \sum_r \pi_k\theta_r[u_1(y-q_{ij},N_k q_{kr}/N_i) - u_2(y-q_{ij},N_k q_{kr}/N_i)N_k q_{kr}/N_i q_{ij} z_r] \tag{32}$$

[6]The special assumption $N_1/N_2 = z_1/z_2$ is needed because in arbitrary discrete sample spaces an observation on a ratio is in general equivalent to an observation on both the numerator and the denominator. Lucas avoids the equivalence by working with continuous sample spaces.

while (31) and the relevant constraints at equality give us one condition

(33) $$0 = \sum_i \sum_k \sum_r \rho_i \pi_k \theta_r [u_1(y-q_{ii}, N_k q_{kr}/N_i) - u_2(y-q_{ii}, N_k q_{kr}/N_i) N_k q_{kr}/N_i q_{ii} z_r].$$

Together, (32) and (33) are three equations in the three values of q_{ij}: q_{12}, q_{21}, and $q_{11} = q_{22}$.

4.2. *A Numerical Example*

I will display the incomplete information solution for the following example: $y = 1.0$, $u[c^h(t)] = (c_1^h)^{1/2} + (c_2^h)^{1/2}$, $z_1 = 1.0$, $z_2 = 1.1$, and $\pi_k = \theta_r = 1/2$ for all k and r.

For these parameter choices, (32) can be written as

(34) $$q_{ij}(y-q_{ij})^{-1/2} = (.25) \sum_k \sum_r [(N_k q_{kr}/N_i)^{1/2}/z_r], \quad i \neq j$$

while (33) can be written as

(35) $$q_{ii}(y-q_{ii})^{-1/2} = (.125) \sum_i \sum_k \sum_r [(N_k q_{kr}/N_i)^{1/2}/z_r].$$

The solution for the q_{ij}'s, which satisfies the hypothesized correspondence between $p(t)$ and $N(t)/z(t)$, is as shown in Table 1. In a long time series for this economy, each of these cells turns up one-quarter of the time. Therefore, observations on the pairs $[q(t), z(t)]$—per capita real saving or real money holdings and the gross growth rate of the money supply—give rise to the scatter diagram shown as dots (●) in Figure 4. (Each dot represents one-quarter of the observations.)

Table 1

N \ z	$z_1 = 1.0$	$z_2 = 1.1$
N_1	.477	.485
N_2	.469	.477

I said above that Lucas' model is one of noninvariant time series relationships. In this context, *noninvariant* means that the distribution of $q(t)$ implied by a different distribution for $z(t)$ cannot be inferred by simple extrapolation from the dots in Figure 4.

Thus, for the same economy but $z_i' = (1.2)z_i$, $i=1, 2$, which represents a more expansionary fiscal policy, the solution for the q_{ij}'s is as shown in Table 2. The implied $[q(t), z'(t)]$ time series scatter is represented by triangles (▲) in Figure 4.

I can also describe the equilibria for some degenerate $z(t)$ distributions. In such cases, the equilibrium is given by the complete information model since if $z(t)$ is nonrandom, an observation on $p(t)$ is equivalent to an observation on

Table 2

N \ z'	$z'_1 = 1.2$	$z'_2 = 1.32$
N_1	.388	.395
N_2	.381	.388

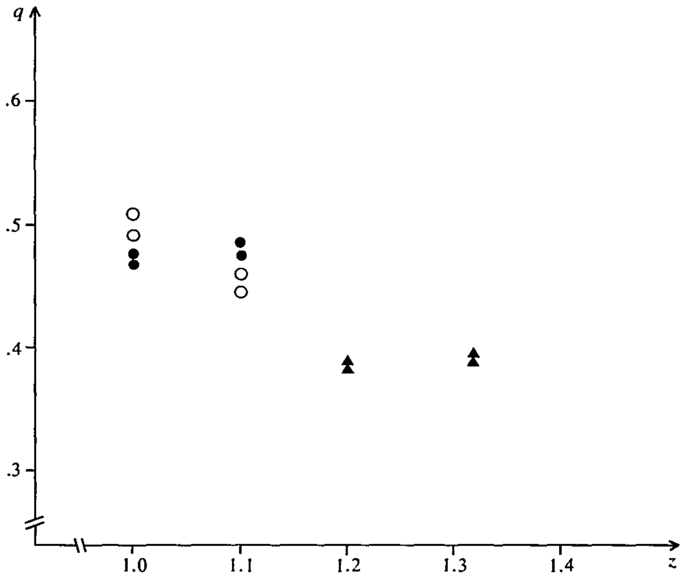
$N(t)$. For the economy of the numerical example, but $z(t) = z$ for all t, (27) can be written as

$$q_i(y-q_i)^{-1/2} = (\eta/2)\sum_j(q_jN_j/N_i)^{1/2}, \quad i=1,2. \tag{36}$$

If $z = 1$ ($\eta=1$), then $q(t) = .508$ when $N(t) = N_1$ and $q(t) = .492$ when $N(t) = N_2$. This solution and that for $z = 1.1$ ($\eta=1/1.1$) are shown in Figure 4 as circles (○).

As these examples demonstrate, the regression equation between $q(t)$ and $z(t)$ that shows up for realizations under a given distribution for $z(t)$ is not one that continues to hold if that distribution is replaced by another distribution. The same is true of regressions of $q(t)$ on $p(t+1)/p(t)$ or on $p(t)/p(t-1)$. Being

Figure 4
Alternative $[q(t), z(t)]$ Time Series Distributions



regressions of a choice variable on a rate of return, one may be tempted to call these *demand functions*. But this would be a mistake, as the reader can verify by studying the distributions of these variables in the examples. This is not to suggest, though, that observations on $[q(t),z(t)]$ are useless. If everything about the economy of the example were known except for the value of γ in $u(c^h) = (c_1^h)^\gamma + (c_2^h)^\gamma$ and the distribution of $z(t)$, then observations on $[q(t),z(t)]$ would permit estimation of γ. This, in turn, would allow predictions to be made about outcomes under different $z(t)$ distributions.

4.3. *General Implications*

Two features of Lucas' model should be kept distinct. One is the nature of the relationships implied under a particular regime. The other is the absence of invariance of these relationships.

With regard to the first feature, Lucas produces a model in which aggregate output is higher the higher is the realized deficit, $z(t)$. This is accomplished by generalizing the model above in two directions. One involves endowing the young with productive, nonstorable labor instead of output and making leisure an additional argument in the utility function. With gross substitution between first- and second-period utility, a positive correlation between per capita output and $z(t)$ is implied.[7] This correlation is converted to one between aggregate output and $z(t)$ by treating the aggregate as a sum over separate subeconomies, each of which is like the economy just described. The subeconomies are related in two ways: at any date t their $N(t)$ realizations sum to a constant and their $z(t)$ realizations are identical.

With regard to the second feature, noninvariance of price-quantity time series correlations is a general property of models in which individuals face uncertainty. In such models, individuals act on the basis of the distributions they think they face.[8] They respond to observations on prices only to the extent that these are relevant as conditioning variables for subsequent distributions they will face. In particular, the individual terms in a time series of prices and quantities implied by such a model are not the outcomes of a sequence of separate conceptual experiments in which individuals are faced with alternative nonstochastic budget sets. Yet despite the fact that macroeconomics is concerned almost entirely with saving and portfolio decisions in which uncertainty and dynamics are acknowledged to play crucial roles, macroeconometric models and, indeed, the macroeconomic paradigm that flows from Keynes' (1936) *General Theory* via Hicks (1937) consist of nothing more than interpreting each term in a time series as the outcome of a separate, static, nonstochastic experiment.

As this discussion suggests, one does not need anything as subtle as Lucas' incomplete information model in order to produce noninvariant time series correlations. The role of the incomplete information model is to produce positive correlations between variables like the government deficit and total output. It is worth noting that Keynes defined the existence of involuntary unemployment as no more than the existence of such Phillips curve correlations.[9]

[7]See Lucas 1977 for a defense of the assumed labor supply response.

[8]One well-known example is the Tobin (1958) portfolio model. For other examples, see Lucas 1976.

[9]According to Keynes (1936, p. 15), people are "involuntarily unemployed if, in the event of a small rise in the price of wage-goods relatively to the money-wage, both the aggregate supply of

5. The Insignificance of Open Market Operations

In the models so far examined, there is a close association between the time path of the supply of fiat money and the time path of its value. Moreover, the usual neutrality proposition holds for those models in the following sense. If $[\hat{c}(t-1),\hat{k}(t),\hat{p}(t)]$ for $t=1, 2, \ldots$ is an equilibrium for a given $M(t)$ sequence, then $[\hat{c}(t-1),\hat{k}(t),\hat{p}(t)/\lambda]$ is an equilibrium for the sequence $\lambda M(t)$ for any $\lambda > 0$.

But those results imply nothing about the effects of changes in the money supply that are brought about through asset exchanges. For asset exchanges, it will be shown that the following is true: If fiscal policy is held fixed (in a way to be made precise below) and government and private transaction costs are the same, then the portfolio of the government—that of a consolidated Federal Reserve–Treasury, say—does not matter, even for the value of fiat money.[10]

The need for a proviso about fiscal policy is clear. If the Federal Reserve makes a loan or buys a security that pays interest, some assumption must be made about what happens to the interest. My assumption corresponds to assuming that the interest goes to the Treasury which uses it to reduce other taxes or to increase other transfers.

I will first ignore transaction costs and consider a model in which the asset purchased in an open market operation is real capital or, equivalently, a title to it. I will then turn to the special case in which open market operations are conducted in government bonds. I will close the section by arguing that there is no obvious evidence that contradicts the view that asset exchanges hardly matter.

5.1. *Government Holdings of Risky Real Assets*

I will work with the model of section 2.2 except that here x, the return on storage, is in each period an independent drawing from a discrete probability distribution: $x(t) = x_i > 0$ with probability $\pi_i > 0$, where $i=1, 2, \ldots, I$ and where $\Sigma_i \pi_i x_i > n$. Moreover, the realization of $x(t)$—the gross return on goods stored from t to $t+1$—occurs after generation $t-1$ disappears and after generation t commits itself to a portfolio, but before generation $t+1$ appears. Thus, intergeneration risk sharing via markets is precluded.

I will also assume that the common utility function u is von Neumann–Morgenstern and that each young person maximizes expected utility. I continue to assume $N(t)/N(t-1) = n$.

5.1.1. *Possible Stationary Equilibria With a Fixed Money Supply and No Government Storage*

It is convenient to describe the choice problem and, hence, equilibrium conditions in terms of demands and supplies for contingent claims on second-period consumption. Thus, let s_i be the price in units of first-period consumption of one unit of second-period consumption in state i, state i being the outcome $x(t) = x_i$. I will denote by c_{2i} second-period consumption in state i.

labour willing to work for the current money-wage and the aggregate demand for it at that wage would be greater than the existing volume of employment."

[10] It may be helpful to note that this is nothing but a Modigliani-Miller irrelevance-of-financial-structure result in which the government is a financial intermediary in the same way as is the corporation in the Modigliani-Miller theory.

Each young person chooses c_1 and c_{2i} for each i to maximize $\Sigma_i \pi_i u(c_1, c_{2i})$ subject to

$$c_1 + \sum_i s_i c_{2i} \leqslant y \tag{37}$$

treating the s_i's as parameters. The necessary and sufficient conditions for a maximum are (37) at equality and for each i

$$\pi_i u_2(c_1, c_{2i}) = s_i \sum_{j=1}^{I} \pi_j u_1(c_1, c_{2j}). \tag{38}$$

These conditions are implicit demand functions.

Supplies are generated by competitive firms. Profits from storing $k \geqslant 0$ units of consumption good are $k(\Sigma_i s_i x_i - 1)$ and must be nonpositive in any competitive equilibrium. Thus, $\Sigma_i s_i x_i = 1$ in an equilibrium with positive storage. Profits from storing m units of money are $m[p(t+1)\Sigma_i s_i - p(t)]$, it being taken for granted that there is an equilibrium in which $p(t+1)$ does not depend on i. Moreover, anticipating that in a stationary monetary equilibrium with a fixed money supply $p(t+1) = np(t)$, it follows that $n\Sigma_i s_i = 1$ in such an equilibrium.

Thus, a stationary nonmonetary equilibrium for this economy consists of nonnegative values of k, c_1, and c_{2i} and s_i for each i that satisfy (37) at equality, (38), the nonpositive profit condition, $\Sigma_i s_i x_i \leqslant 1$, and

$$c_{2i} = x_i k, \quad i = 1, 2, \ldots, I \tag{39}$$

where k is per capita storage of the consumption good. It is easy to show that such an equilibrium with $k > 0$ exists and is unique. We denote the equilibrium values by adding a star (*).

A stationary monetary equilibrium for this economy consists of nonnegative values of k, c_1, c_{2i}, and s_i for each i and a positive value of q that satisfy (37) at equality, (38), two nonpositive profit conditions, $\Sigma_i s_i x_i \leqslant 1$ and $n\Sigma_i s_i = 1$, and

$$c_{2i} = x_i k + nq, \quad i = 1, 2, \ldots, I \tag{40}$$

where $q = p(t)M/N(t)$ for all t. I will denote any such equilibrium by adding two stars (**).[11]

5.1.2. *Stationary Monetary Equilibria With Government Storage*

Let k_g be per capita storage by the government. I will study equilibria in which the government acquires k_g by a new issue of money and in which it maintains these holdings in every period but does not alter its consumption because it is storing k_g. This implies that the government must transfer the amount $x(t)N(t)k_g - N(t+1)k_g$ in period $t+1$. I assume that each of the $N(t)$ members of generation t receives $(x_i - n)k_g$ if $x(t) = x_i$ in the second period of her or his life. Moreover, this transfer (or tax, if negative) is viewed as lump-sum. These assumptions imply

PROPOSITION 8. *If the stationary monetary equilibrium exists, then the contingent claims prices, the consumption allocation, and the value-of-money*

[11] The condition $\Sigma_i \pi_i x_i > n$ insures that $K^{**} > 0$ in any stationary monetary equilibrium.

sequence of that equilibrium, and the per capita portfolio, $k = k^{**} - k_g$ *and* $q = q^{**} + k_g$*, are an equilibrium for any* $k_g \leq k^{**}$.

Proof. At $s_i = s_i^{**}$ for all i, the value of taxes $k_g \Sigma_i s_i (x_i - n)$ is zero. Thus, (37) remains the budget set, while in place of (40), we have

$$c_{2i} = x_i k + nq + (x_i - n)k_g. \tag{41}$$

Thus, if $k = k^{**} - k_g$ and $q = q^{**} + k_g$, then $c_{2i} = c_{2i}^{**}$. To verify that the $p(t)^{**}$ sequence is an equilibrium value-of-fiat-money sequence, we must show that for this price sequence, the amount of money, M, that satisfies the government budget constraint for the purchase of k_g, namely,

$$p(t)^{**}(M - M^{**}) = N(t)k_g \tag{42}$$

also satisfies $q = q^{**} + k_g$. But this is immediate and completes the proof.

Proposition 8 is not a liquidity-trap result. It is not the case that private sector demands for storage and money holdings are perfectly elastic. The result arises because of the way profits and losses on the government's portfolio are passed back to individuals by way of transfers and taxes.

It is crucial for Proposition 8 that taxes and transfers be passed back on an equal per capita basis. Thus, for example, if instead, any taxes ($x_i < n$) are levied on, say, the odd-numbered members of generation t and any transfers ($x_i > n$) are given to the even-numbered members of generation t, then Proposition 8 does not hold. It is not necessary, however, that individuals be identical in tastes and endowments.

The upper bound on k_g is also crucial. If government storage exceeds what would otherwise be undertaken by the private sector in a monetary equilibrium, ($k^{**} < k_g \leq k^*$), then since private storage is nonnegative, private holdings cannot simply offset government holdings. The larger is k_g in this range, the closer is the economy pushed toward a nonmonetary equilibrium. In other words, in this range the more the government stores, the lower is the value of fiat money.[12] Whether this is to be regarded as monetary policy is debatable; the equilibrium value of transfers implied by unchanged government consumption is negative if $k_g > k^{**}$.

5.2. *Government Bonds and the Optimal Financing of Deficits*[13]

In the United States, open market operations are conducted mainly in government bonds. For a given fiscal policy, one may approach the study of such open market operations by posing the following question: For a given time path of total (outside) government debt, the time path implied by fiscal policy, what are the effects of varying the proportions of the total between fiat money and so-called interest-bearing debt?

Any theory of the composition of total government debt must explain positive interest on safe government debt, where *safe* means free of default risk. Bryant and Wallace (1979a) argue that transaction costs must be invoked to explain positive nominal interest on such debt. One transaction-cost

[12] To find a stationary monetary equilibrium for $k_g \in (k^{**}, k^*)$, solve (37) at equality with y replaced by $y + k_g \Sigma_i s_i (x_i - n)$, (38), and (41) with $k = 0$ for c_1, q, c_{2i}, and s_i with $\Sigma_i s_i = 1/n$ and $\Sigma_i s_i x_i \leq 1$. Then the value of money, $p(t)$, is found from $q - p(t)M^{**}/N(t) = k_g$ and the money supply, M, from $q = p(t)M$.

[13] This section summarizes Bryant and Wallace 1979a,b.

model posits that government bonds (Treasury bills and so forth) are issued in denominations so large that most potential holders cannot or do not want to hold them directly. They must be broken up or intermediated by way of a resource-using intermediation technology.[14] But the detailed model of transaction costs is probably less important than the recognition that if safe bonds bear nominal interest, then it must be that the bonds are somehow more costly for the private sector to absorb than is fiat money.

By itself, nothing is implied for the optimal composition of total debt from such relative costliness of bonds. If the private sector costs of absorbing bonds are precisely matched by government real resource savings from issuing bonds (instead of issuing fiat money), then a version of Proposition 8 would hold; that is, the composition of the total of net or outside debt would not matter. Bryant and Wallace do not assume such symmetry of transaction costs. Instead, they assume that the government—a consolidated Treasury–Federal Reserve in the United States — is indifferent in terms of real resource costs between issuing fiat money and issuing and selling bonds.

This particular asymmetry implies a very simple result. Barring special circumstances having to do with price discrimination or particular second-best situations, if bonds bear interest, then too many are outstanding. Indeed, if bonds bear interest whenever the public holds any, then — and again barring special circumstances — the only efficient composition of total outside debt is one composed entirely of fiat money.

That the possibility of price discrimination could vitiate this result is obvious. If by issuing different kinds of debt, the government, a monopolist in issuing safe titles to fiat money, can face groups with different demand functions for nominally safe debt with different rates of return, then efficiency does not call for all debt to be zero-interest debt. Recall that perfect price discrimination is consistent with efficiency.

One potentially relevant second-best situation concerns the role of currency reserve requirements in limiting the size of insured intermediation. (See Bryant and Wallace 1979b.) Given a system of improperly priced deposit insurance—and, on the face of it, both that provided by the FDIC and that provided by Federal Reserve lender-of-last-resort activity would seem to be improperly priced — portfolio restrictions, including reserve requirements, play a potentially important role in limiting the effects of the distorting incentives produced by the insurance.

But neither of these special circumstances can be expected to validate the usual analysis of open market operations. From a macroeconomic point of view, there is a sense in which such special circumstances — including the sort of transaction-cost asymmetry assumed by Bryant and Wallace — are only wrinkles on Proposition 8. The theoretical presumption is that the value of fiat money is not much affected by the composition of the total of the net or outside debt of the government.

5.3. *Casual Remarks on Evidence*

As with the absence-of-dominance implication, the implication that the composition of the government's portfolio is largely irrelevant for the value of fiat money cannot be dismissed easily by casual empiricism.

[14]To invoke transaction costs to explain positive nominal interest on safe assets is not new. This is done in the inventory models of money demand. See Baumol 1952, Tobin 1956, and Miller and Orr 1966.

While there are many historical episodes in which measures of money and measures of total nominal income and/or the price level move up and down together, the implied simple correlations by themselves say nothing about the effects of government asset exchanges.

Most, if not all, of these episodes are ones in which increases in the amount of money are accompanied by asset creation and decreases by asset destruction. Throughout history, the major increases in the amount of money have come about by way of coinage debasements, discoveries of the commodity money, and budget deficits; the major decreases have come about by way of banking panics and, to a lesser extent, budget surpluses. None of these qualify as asset exchanges. Moreover, from the point of view of the models described above, such changes in the amount of money ought to be accompanied by price level changes.

Some would take the implications of macroeconometric models to be a second source of evidence against the view that asset exchanges do not matter. But this would be a mistake. As has long been recognized, the implications for monetary policy that flow from such models depend in a crucial way on the estimated so-called money-demand functions imbedded in the models. But these estimated relationships consist of little more than the simple correlations between measures of money and measures of total nominal income referred to above. And like them, they may well take the form they do only because they are estimated on observations generated by policies under which changes in the measure of money are accompanied by changes in asset totals in the same direction.

A third purported body of evidence is designed to overcome the objection just raised to simple correlation studies. It consists of time series regressions of variables like the price level, total nominal income, real income, and the unemployment rate on measures of both monetary policy and fiscal policy and perhaps some other variables. Examples are recent studies by Stein (1976) and Perry (1978). These investigators treat their regression equations as invariant relationships, as relationships that would continue to hold no matter what rules for the monetary and fiscal policy variables were put into effect. For a host of reasons, many of which already appear in the literature, such a view is preposterous. Yet these authors proceed as if invariance is obvious and does not even require a defense.

I will cite just a few reasons why the invariance view of such regression equations is farfetched. In any sensible model, the correlation (simple or partial) that turns up in time series between a measure of the deficit and other variables under a policy regime in which the deficit is in part random says little about what the response of the economy would be to alternative permanent levels of the deficit. (This is true in both the complete and incomplete information versions of the model described in section 4.) One reason is that a deficit viewed as temporary and likely to be offset by some future surplus gives rise to quite different expectations about future tax liabilities and future prices than does a new level of the deficit that is viewed as permanent. In a similar vein, can it be that the regression that turns up is independent of the sort of policy that the Federal Reserve was following? In particular, can it be independent of whether the Federal Reserve was focusing on interest rates or monetary aggregates? Finally, the invariance view of these regressions implies that nothing happens at time t if it is then announced that a huge permanent deficit will be run starting at time $t+1$. Is that believable?

Unfortunately, I am not clever enough to end these remarks with a description of the right set of tests of the view that the government's portfolio hardly matters. It is even conceivable that the relevant experiment has not yet been performed![15]

6. Country-Specific Fiat Moneys[16]
So far I have considered economies with one government and at most one fiat money. I now turn to a world economy with many governments and, at least potentially, many fiat moneys. The main result is that the market left to itself cannot cope with more than one fiat money.

Consider two economies (countries) of the kind described in section 2.2. For simplicity, let these be identical except that each country has its own tax-transfer scheme. (Think of country 1 as issuing red pieces of paper and country 2 green pieces of paper.) If residents the world over are free to choose to hold whatever money they want, then it follows that if there is an equilibrium in which at least one money has value, then the value of one relative to that of the other (the exchange rate) is constant over time. This is an instance of the absence-of-dominance implication. Moreover, for any unchanging exchange rate, there are equilibria. Why? Name an exchange rate. This allows us to aggregate the red and green pieces at any date into an equivalent number of red pieces, say. Put differently, the named exchange rate and the time paths of the separate money supplies imply a time path of the world money supply. And for any such path, we can display the possible equilibria. Since this is true for any named exchange rate, it follows that there is no market-determined exchange rate. Nor can this result be avoided by simply adding uncertainty. The indeterminacy will not go away by making the deficit in one or both of the countries random.

The indeterminacy just described arises in a regime in which there is no intervention in exchange markets and in which there is unfettered international borrowing and lending; that is, there are no capital controls. It is a regime in which each government does no more than operate its own tax-transfer scheme. Kareken and Wallace (1978) argue that only two kinds of regimes can be expected to prevail. If governments insist on autonomy of budget policies — different values of z in the context of the model of section 2.2 — then the relative values of the two currencies must change over time with that of the country with the higher value of z falling relative to that of the other country, and there must be controls that prevent residents of one country from freely borrowing and lending internationally. But if governments coordinate budget policies—have identical z's—then there can be unfettered international borrowing and lending if the exchange rate is fixed either cooperatively or by one country acting on its own.

These results are established in two steps. The first is a standard argument. If the budget policies differ, then the exchange rate cannot be constant. If it were, residents of the country with the smaller deficit (smaller value of z) would eventually be permanently subsidizing residents of the other country. [To argue that there is no subsidy if the country with the smaller deficit holds

[15]Surely one cannot take the view that history has necessarily generated enough observations to allow us to decisively test every proposition of interest. If this is true for propositions in economics, why not also for those in every other science?

[16]This section consists of a brief summary of Kareken and Wallace 1978.

its reserves in the form of interest-bearing assets is fallacious since it depends on the Fisherian notion that (most?) real interest rates do not depend on the anticipated inflation rate.] The second step simply invokes absence of dominance. The exchange rate can change only if residents are restricted in their portfolio choices.

7. Concluding Remarks

Stanley Fischer (1975, p. 159) concluded the section of his survey paper devoted to microeconomic foundations of money as follows:

> This work [that on the foundations of monetary theory] is obviously both difficult and only a beginning. It is not clear where, if anywhere, it will lead. It will no doubt provide more convincing and carefully worked out explanations for the use of a medium of exchange than we now have, but it appears that those explanations will not be fundamentally different from the traditional verbal explanations, and that they will not have any major consequences for the way in which macromodels are built.

Fischer's pessimism is due, in part, to the fact that the work he surveyed was not carried to the point where it could confront most practical problems in monetary theory and policy and, in part, to the notion that the only practical implications that could possibly flow from such work are a suggested list of arguments and a functional form for the demand function for money.

From our vantage point, the pessimism seems unwarranted. While the reader may find much to quarrel with in what I have presented, two points must be conceded. First, models built on the overlapping generations friction do meet the criteria for being models of money with a microeconomic foundation. (In slightly different words, those models do successfully integrate value and monetary theory.) Second, they do confront and have radical implications for practical issues in monetary theory and policy. Moreover, I would hazard two conjectures, the first with more confidence than the second. Any microeconomic model of money that maintains intrinsic uselessness will have implications sharply at variance with currently held views. Furthermore, any such model will have implications not all that different from those described in the body of this paper.

Yet many readers, I surmise, remain unconvinced. Many, I suspect, still hold to the view that the overlapping generations model is defective because it captures only the *store-of-value* function of money and, in particular, does not capture the *medium-of-exchange* function of money.

Presumably, the medium-of-exchange function is meant to be an additional function of money. (Since money must be held in any model of it, any model of money displays the store-of-value function.) But what is this additional medium-of-exchange function? One hopes that giving money this additional role is not the same as abandoning intrinsic uselessness and is not tantamount to requiring that we find a role for money in a nondynamic setting. But if not, then what does it mean? To say that money plays a medium-of-exchange role if it facilitates trade or is traded relatively frequently is not very helpful. After all, money gets traded in the overlapping generations model; in some versions it is the only thing traded, in others not. And, of course, it facilitates trade; in some versions, the old and the young cannot trade at all except via money. Could it be that this distinction between roles of money is not all that useful? The possibility should be taken seriously. After all, although this distinction has been around for a long time, not a single proposition in monetary theory makes use of it. At a minimum, this should make us question whether saying that a model gives money this or that role is a serious criticism of the model.

But none of this is to suggest that the overlapping generations model is, as it were, the last word in models of money. (I would say it is the first word in models of money.) As noted above, there is general agreement that one needs to introduce some sort of friction into the Walrasian (Arrow-Debreu) general equilibrium model in order to get from it a model of money. But some friction is also needed for a more general problem; the explanation of the form that interactions take among agents where interactions via Walrasian markets is a limiting special case. The obvious goal is a specification of friction that simultaneously provides insights into many forms of interaction, not just market interactions via fiat money.

A related limitation of the overlapping generations model is that money, in a way, works too well in that model. Money completely overcomes the friction. Given a real friction, there is no reason why it should be feasible to overcome it completely.[17] A somewhat trivial way to overcome this defect is to make running the monetary system costly, for example, by making it costly to print money and to prevent counterfeiting. A less trivial way of dealing with this defect involves a theory of the social management of fiat money. Such a theory may also eliminate either the nonmonetary or the monetary equilibrium when both exist. All we have now in the face of these multiple equilibria are the propositions, established for some versions that connect the nonoptimality of the nonmonetary equilibrium with the existence and optimality of some monetary equilibrium.

[17] This is related to if not identical with Hahn's (1973a) distinction between essential and inessential monetary equilibria.

Appendix

A1. Proof of Proposition 2
Letting $\bar{q}(t) = \bar{p}(t)M(t)/N(t)$ for all t, the forecasting scheme (13) is equivalent to

$$\text{(A1)} \qquad \bar{q}(t+1) = \lambda\bar{q}(t) + (1-\lambda)q(t) \quad \text{all } t \geqslant 2.$$

But $q(t)$ as a function of $\bar{q}(t+1)$ is given by

$$\text{(A2)} \qquad q(t) = \begin{cases} H[\bar{q}(t+1)] & \text{if } (n/z)\bar{q}(t+1)/q(t) > x \\ \bar{q}(t+1)(n/zx) & \text{otherwise} \end{cases}$$

where, recall, $\bar{p}(t+1)/p(t) = (n/z)\bar{q}(t+1)/q(t)$ is the predicted rate of return on money.

I will use (A1) and (A2) to find $\bar{q}(t+1)$ as a function $\bar{q}(t)$.

If $q(t)$ is given by the upper branch of (A2), then by (A1)

$$\text{(A3)} \qquad \bar{q}(t+1) = \lambda\bar{q}(t) + (1-\lambda)H[\bar{q}(t+1)].$$

But by the hypothesis on H, $\bar{q}(t)$ and $\bar{q}(t+1)$ satisfy (A3) if and only if

$$\text{(A4)} \qquad \bar{q}(t+1) = G[\bar{q}(t)]$$

where $G' = \lambda/[1-(1-\lambda)H'] \in (0,1)$ and where $\hat{q}$ is the unique fixed point of both H and G. It follows that $\bar{q}$ sequences that satisfy G converge monotonically to $\hat{q}$.

If $q(t)$ is given by the lower branch of (A2), then by (A1)

$$\text{(A5)} \qquad \bar{q}(t+1) = \bar{q}(t)\lambda/[1-(1-\lambda)n/zx].$$

To prove parts a and b of Proposition 2 we consider two cases:

1. $xz/n \leqslant 1$, $\bar{q}(t) \geqslant \hat{q}$. Suppose at least one of the hypotheses holds with strict inequality. We first show that, then, only the upper branch of (A2) is consistent with the hypotheses. To verify consistency, note that $\bar{q}(t) \geqslant \hat{q}$ implies $G[\bar{q}(t)] \geqslant \hat{q}$ which implies $H\{G[\bar{q}(t)]\} \leqslant G[\bar{q}(t)]$ which, in turn, implies satisfaction of the proviso for the upper branch of (A2). To verify inconsistency with the lower branch of (A2), note that (A5) implies $\bar{q}(t+1) > \hat{q}$ which implies choosing greater second-period consumption at the rate of return x than at the rate of return $n/z \geqslant x$, a violation of our normal goods assumption on preferences. Thus, with strict inequality in one of the hypotheses, we have monotone convergence of $\{\bar{q}(t)\}$ to $\hat{q}$ via (A4). If both hypotheses hold with equality, then $\bar{q}(t+1) = \hat{q}$ by (A5).
2. $xz/n < 1$, $\bar{q}(t) < \hat{q}$. Here either branch of (A2) may be consistent with the hypotheses. But whichever is we get $\bar{q}(t+1) > \bar{q}(t)$ and convergence of $\bar{q}$ to $\hat{q}$. [Note that (A5) is exponential increasing with $xz/n < 1$.]

Now since $xz/n \leqslant 1$ implies that $\check{q} = q^*$ and since convergence of $\{\bar{q}(t)\}$ to $\check{q}$ implies convergence of $\{q(t)\}$ to $\check{q}$ cases 1 and 2 give us parts a and b of the proposition.

To prove part c we consider $xz/n = 1$ and $\bar{q}(t) < \check{q}$. Here $\bar{q}(t+1)$ can only be taken from (A5), that is, $q(t)$ and $\bar{q}(t+1)$ can satisfy only the lower branch of (A2). But (A5) implies $\bar{q}(t+1) = \bar{q}(t)$ and, hence, part c of the proposition.

To prove part d we again consider two cases:

1. $\bar{q}(t) > \check{q}$. Here $\bar{q}(t+1)$ and $q(t)$ may be consistent with either branch of (A2). But both imply $\bar{q}(t+1) < \bar{q}(t)$. We cannot get convergence to $\check{q}$ because for $\bar{q}(t)$ close enough to $\check{q}$, $xz/n > 1$ implies that $\bar{q}(t+1)$ and $q(t)$ cannot satisfy the upper branch of (A2). Thus, we eventually get some $\bar{q} < \check{q}$, which brings us to the second case.
2. $\bar{q}(t) \leqslant \check{q}$. Here $\bar{q}(t+1)$ must satisfy (A5) which implies convergence of $\{\bar{q}(t)\}$ to 0 and hence convergence of $q(t)$ to 0, as we set out to prove.

A2. Proof of Proposition 3

Let a caret (ˆ) over a term denote an equilibrium allocation and a bar (¯) over a term a feasible Pareto superior (P.S.) allocation. I will show that the assumed existence of the latter gives rise to a contradiction.

Without loss of generality, assume that the P.S. allocation satisfies (14) with equality and that for all $t \geqslant 1$, $v[\bar{c}^h(t)] = v[\bar{c}^{h'}(t)]$ for all h and h' in generation t. (Given an allocation P.S. to the equilibrium allocation that does not satisfy these conditions, one can easily construct the P.S. allocation that is P.S. to the former and, hence, to the latter.)

I will prove in detail that $\bar{K}(t) = \hat{K}(t)$ for all t and will then refer the reader to published results on pure exchange economies for the rest.

Suppose $\bar{K}(t) \neq \hat{K}(t)$ for some t. Then there is a smallest $t \geqslant 1$ at which this happens. I first rule out

A first departure of the form $\bar{K}(t) > \hat{K}(t)$.

Being a first departure, it follows from (14) with equality that either (a) $\bar{C}_2(t-1) < \hat{C}_2(t-1)$ or (b) $\bar{C}_1(t) < \hat{C}_1(t)$ or both.

Case (a): This is easy. Since t is the first departure of $\{\bar{K}\}$ from $\{\hat{K}\}$, we have for $i = 1, 2, \ldots, t-1$

$$\text{(A6)} \qquad \bar{C}_1(t-i) + \bar{C}_2(t-i-1) = \hat{C}_1(t-i) + \hat{C}_2(t-i-1).$$

But by (a) and the properties of a P.S. allocation, $\bar{C}_1(t-1) > \hat{C}_1(t-1)$. One then proceeds backward from $t-1$ to $t-2$ and so on using (A6) to conclude that $\bar{C}_2(0) < \hat{C}_2(0)$, a contradiction.

Case (b): This is more demanding. Under a feasible P.S. allocation, the members of generation t must have more second-period consumption than under an equilibrium allocation. And since $v[\hat{c}^h(t)] \geqslant x$, the extra storage does not produce enough. Therefore, it follows by (14) at equality that

$$\text{(A7)} \qquad \hat{C}_1(t+1) + \hat{K}(t+1) - [\bar{C}_1(t+1) + \bar{K}(t+1)] \equiv N(t+1)d(t+1) > 0.$$

I now show by induction that the d sequence is increasing and unbounded. Since $[\hat{C}_1(t+1) + \hat{K}(t+1)]/N(t+1) \leqslant y$, this will rule out $\bar{K}(t) > \hat{K}(t)$ under case (b).

For the induction step, we use (A7) as an initial condition and consider the following problem.

Choose $c(t+1)$—an allocation for members of generation $t+1$—to minimize $C_2(t+1)$ subject to

(A8) $$\hat{C}_1(t+1) + \hat{K}(t+1) - [C_1(t+1) + \bar{K}(t+1)] \geq N(t+1)d(t+1)$$

(A9) $$u[c^h(t+1)] \geq u[\hat{c}^h(t+1)].$$

Since $\bar{c}(t+1)$ is feasible for this problem—that is, satisfies (A9) and (A8) [see (A7)]—we have $\bar{C}_2(t+1) \geq \tilde{C}_2(t+1)$, where a tilde ($\tilde{}$) over a term denotes a solution value for this minimization problem. Before we use this inequality, though, we want to derive a convenient expression for $\tilde{C}_2(t+1)$ in terms of $d(t+1)$.

It is easily verified that there is a unique solution to this minimization problem that satisfies (A8) and (A9) with equality and, since $\hat{c}^h(t+1) = \hat{c}^{h'}(t+1)$,†

(A10) $$\hat{c}_1^h(t+1) - \tilde{c}_1^h(t+1) = d(t+1) + \Delta(t+1), \text{ all } h$$

where

$$\Delta(t+1) = [\bar{K}(t+1) - \hat{K}(t+1)]/N(t+1).$$

But in general, along a contour of u, $c_2^h = g(c_1^h)$ where $g' = -u_1/u_2 = -v$ and $g'' > 0$. Therefore, applying the intermediate value theorem to g, we have

(A11) $$g(c_1^h) = g(\hat{c}_1^h) + (\hat{c}_1^h - c_1^h)[-g'(\hat{c}_1^h) + f_{\hat{c}}(\hat{c}_1^h - c_1^h)]$$

where the function $f_{\hat{c}}$, whose argument is $(\hat{c}_1^h - c_1^h)$, is strictly increasing and such that $f_{\hat{c}}(0) = 0$.

Now since $\tilde{c}^h(t)$ and $\hat{c}^h(t)$ are on the same contour of u, we may use (A10) and (A11) to write

(A12) $$\tilde{c}_2^h(t+1) = \hat{c}^h(t+1)$$
$$+ [d(t+1) + \Delta(t+1)]\{v[\hat{c}^h(t+1)] + f_{\hat{c}(t+1)}[d(t+1) + \Delta(t+1)]\}$$

or since

$$\bar{C}_2(t+1) \geq \tilde{C}_2(t+1) = N(t+1)\,\tilde{c}_2^h(t+1),$$

(A13) $$[\bar{C}_2(t+1) - \hat{C}_2(t+1)]/N(t+1)$$
$$\geq [d(t+1) + \Delta(t+1)]\{v[\hat{c}^h(t+1)] + f_{\hat{c}(t+1)}[d(t+1) + \Delta(t+1)]\}.$$

But since the P.S. and equilibrium allocations satisfy (14) at equality, we have

$$\hat{C}_1(t+2) + \hat{K}(t+2) - [\bar{C}_1(t+2) + \bar{K}(t+2)] \equiv N(t+2)d(t+2)$$

$$= \bar{C}_2(t+1) - \hat{C}_2(t+1) - xN(t+1)\Delta(t+1)$$

†We could get by without this last assumption. See Kareken and Wallace 1977.

or

$$\text{(A14)} \qquad d(t+2) = [\bar{C}_2(t+1) - \hat{C}_2(t+1)] / N(t+1)n - x\Delta(t+1)/n.$$

Then using (A13)

$$\text{(A15)} \qquad d(t+2) \geqslant d(t+1)\, v[\hat{c}^h(t+1)]/n + \Delta(t+1)\,\{v[\hat{c}^h(t+1)] - x\}/n$$
$$+ [d(t+1) + \Delta(t+1)] f_{\hat{c}(t+1)} [d(t+1) + \Delta(t+1)]/n.$$

The right-hand side consists of a sum of three terms. The last term has the form $(\cdot)f(\cdot)/n$ which is nonnegative by the properties of f. The second term is also nonnegative since $v[\hat{c}^h(t+1)] \geqslant x$ with strict equality if $\Delta(t+1) < 0$. [If $\Delta(t+1) < 0$, then $\hat{K}(t+1) > 0$.] Thus (A15) implies

$$\text{(A16)} \qquad d(t+2) \geqslant d(t+1)\, v[\hat{c}^h(t+1)]/n \geqslant d(t+1)\,(x/n).$$

Thus the d sequence is bounded below by a strictly increasing exponential and hence is unbounded.

Next I quickly rule out

A first departure of the form $\bar{K}(t) < \hat{K}(t)$.

If there is such a P.S. allocation, then by (14) either

$$\text{(A17)} \qquad \bar{C}_1(t+1) + \bar{K}(t+1) < \hat{C}_1(t+1) + \hat{K}(t+1)$$

or

$$\text{(A18)} \qquad \bar{C}_2(t) < \hat{C}_2(t)$$

or both. If (A17) holds, we have an initial condition for the induction proof just given. If (A18) holds, then $\bar{C}_1(t)$ must exceed $\hat{C}_1(t)$ by more than $x[\hat{K}(t) - \bar{K}(t)]$ because $\bar{K}(t) < \hat{K}(t)$ implies $\hat{K}(t) > 0$ and hence $v[\hat{c}^h(t)] = x$. But then we can work backward as under case (a) above.

We have now proven that if there is a P.S. allocation, $\bar{K}(t) = \hat{K}(t)$ for all t. Therefore, by (14) at equality, any such P.S. allocation satisfies

$$\text{(A19)} \qquad \bar{C}_1(t) + \bar{C}_2(t-1) = \hat{C}_1(t) + \hat{C}_2(t-1) \quad \text{for all } t \geqslant 1.$$

Then, since $v[\hat{c}^h(c)] \geqslant x > n$, one can derive a contradiction from assuming that somebody is strictly better off under a P.S. allocation than under an equilibrium allocation. To prove this the reader can either adapt the case (a) and (b) arguments above or can consult the proof in Kareken and Wallace 1977, which itself is similar to the case (a) and (b) arguments made above.

A3. Proof of Proposition 5

Here I outline a proof that follows closely the proof of Proposition 3.

First, one rules out the existence of a feasible P.S. allocation with a first date $t \geqslant 1$ at which $\bar{K}(t) > 0$. In such a proof one gets to an expression like (A15), but in this case with $\Delta(t+1) \geqslant 0$. And since $v(\hat{c}^h) = n$, one must use the fact that the relevant f function in the third term on the right-hand side of (A15) is strictly increasing. Then, any feasible P.S. allocation—that is, any bar allocation—satisfies (A19) and one can proceed as indicated there.